

Pre 1.2

Number Systems Part II

Lesson Summary: In this lesson, students access a website and pretend to visit another planet where dogs are intelligent and they use base 4. Then they play a game in which they must think like a computer using base 2. Third, they problem solve to figure out different number system challenges.

Utah Core Indicators: Pre-Algebra Content Standard 1.5 Process Standards 1, 3, 4

Broad Understanding:

- Understanding number systems is central to using mathematics. Our base 10 number system utilizes place value and zero. By studying other number systems we can develop greater understanding of how our number system works.

Essential Questions:

- Why are there other number bases?
- How do we add, subtract, multiply and divide in Base 4?
- Why do computers use Base 2?
- What are some benefits of using Base 10?

Knowledge and Skills:

- Problem Solving
- Number bases and place value as related to base 10

Assessment Evidence:

- Evaluate the Base 4, Think like a Computer, and Alien Number Systems for completion and quality.
- Students will write a Summary of Learning for the Number Systems activities. This should include 1) learning from activities, 2) ideas gained from large group discussions, 3) responses to the essential questions. Score using the Communication and/or Understanding rubrics.
- Students will dive underwater to a world run by a clever octopus that uses Base 8. They should demonstrate their understanding of bases and place value by demonstrating what the clever octopus told them about base 8. Evaluate using

Learning Plan

Materials: calculators, computers

Time: 2-5 days

Lesson Type: Inquiry and Problem Solving, Use cooperative groups or teams

Directions:

1.2a Number Bases (Base 4) Access the activity on the following website.

<http://www.dsgb.org.uk/doggie.htm>

Discourse Suggestions:

- Imagine we are one another planet ruled by intelligent canines.
- When we count 1, 2, 3, 10...what do we call the "10"?
- When we get to 31, 32, 33...then what?

1.2b Think like a Computer and Base 2 Game. Students will learn how to think like a computer and play a Base Two game. See below.

Teacher Talk - Think like a Computer

Our number system is based on the number 10. How far can we count until we need to use two place values? How far can we count until we need to use three place values? How many times larger is each place value than the previous one?

1,000,000	100,000	10,000	1,000	100	10	1
one million	hundred thousand	ten thousand	one thousand	hundred	ten	one

In base four how far could we get until we had to use two place values? How many times bigger do you think each place value is in base 4?

How far can we get before we have to use two place values in base 2?

Today we are going to think like a computer. Remember computers run on electrical power. This means that they can be ON or OFF. Computers work with a number system called Base Two. These two bases are either ON or OFF. Let's draw the place value on the board. The first place is a 1 (the same as our base 10.) Now instead of multiplying by 1- as in base 10, we multiply by 2 because this is Base 2. So the second place is 2 times the first place (1) which = 2. The third place is 2 times the second place (2) which = 4. Continue across the places multiplying by 2.

64 32 16 8 4 2 1

When working with Base 10 we only use the digits 0 – 9 (we don't use 10)
When working with Base 2 we only use the digits 0 – 1 (we don't use 2)

The 0 represents OFF and the 1 represents ON
So the computer is either ON or OFF.

Activity Directions – Think like a Computer

1. Pass out the "Student Worksheet – Think like a Computer."
2. Remind students that the place values are 1, 2, 4, 8, 16, 32, 64, 128, and etc.
3. Put a 1 in the first column. This is our Base 10 symbol for one.
Put an x in the second column. This shows a group of one.
Put a 1 in the third column. This is the Base 2 symbol for one.
4. Put a 2 in the first column. This is our Base 10 symbol for two.
Put two x's in the second column. This shows a group of two. (circle the group)
Put a 10 in the third column. This is the Base 2 symbol for two.
The 1 means we have one group of two.
The 0 means we have no groups of one.
5. Put a 3 in the first column. This is our Base 10 symbol for three.
Put two three x's in the second column. This shows a group of two and one. (circle the group of two)
Put 11 in the third column. This is our Base 2 symbol for three.
The 1 means we have one group of two.

- The other 1 means we have one group of one.
6. Put a 4 in the first column. This is our Base 10 symbol for four.
Put four x's in the second column. This shows a group of four. (circle the group of four)
Put a 100 in the third column. This is the Base 2 symbol for four.
The 1 means we have one group of four.
The first 0 means we have no groups of two.
The next 0 means we have no groups of one.
 7. Put a 5 in the first column. This is our Base 10 symbol for five.
Put five x's in the second column. This shows a group of four and one. (circle the group of four)
Put a 101 in the third column. This is the Base 2 symbol for five.
The first 1 means we have one group of four.
The 0 means we have no groups of two
The other 1 means we have one group of one.
 8. Have students continue on through the number 10. Then have the students discuss how 11 in Base 10 would look like in the "Grouping" column and in the "Base 2 Symbol" column.
The students should have eleven x's with a group of eight circled, a group of two circled and one remaining x under the "Grouping" column. Under the "Base 2 Symbol" column they should have 1011. The first 1 means they have a group of eight. The 0 means they have no groups of four. The second 1 means they have a group of two. The third 1 means they have a group of one.
 9. When the class has finished their discussion for 11 in Base 10, have them complete their chart to 16 in Base 10.
 10. Have students write a paragraph describing any patterns that they see.

Base 2 Game Directions

The teacher makes up a number. Students place beans on the game boards (see below) to make the number into base 2. Use the base 2 place value chart (see below) if needed.

1.2c Alien Number Systems. This problem solving activity is from United We Solve (Erickson). Students work in groups of three or four and are given clues. They must work together to solve the alien number system. (See below)

Example of chart students will create

<i>Base 10 Symbol</i>	<i>Grouping</i>	<i>Base 2 Symbol</i>
1	x	1 ₂
2	xx	10 ₂
3	xx x	11 ₂
4	xxxx	100
5	xxxx x	101
6	xxxx xx	110
7	xxxx xx x	111
8	xxxxxxxx	1000
9	xxxxxxxx x	1001
10	xxxxxxxx xx	1010
11	xxxxxxxx xx x	1011
12	xxxxxxxx xxxx	1100
13	xxxxxxxx xxxx x	1101
14	xxxxxxxx xxxx xx	1110
15	xxxxxxxx xxxx xx x	1111
16	xxxxxxxxxxxxxxxxxxxx	10000

Play the Base 2 Game—Record the made up numbers and then the base 2 numbers.

Name _____

Date _____

For Fun Let's Add Like a Computer

Develop a method of how you would add the following Base 2 numbers. DO NOT use Base 10 numbers in this method.

(Hint: $1 + 1 = 10$)

$$\begin{array}{r} \text{Add: } 101 \\ \quad \underline{111} \end{array}$$

How can you prove that you are correct?

(Hint: You may want to use your knowledge of Base 10 numbers to help you.)

Try your method and see if it works on the next problem.

$$\begin{array}{r} \text{Add: } 1001 \\ \quad \underline{101} \end{array}$$

Discuss with another group how you add using Base 2. Write a paragraph describing the other group's method and the similarities and differences between the two methods.

Decide which method you like best and add the following.

$$\begin{array}{r} \text{Add: } 111 \\ \quad \underline{11} \end{array} \qquad \begin{array}{r} 1100 \\ \quad \underline{10} \end{array} \qquad \begin{array}{r} 111 \\ 100 \\ \underline{110} \end{array}$$

Extension:

Devise a method for multiplying with Base 2 numbers.

8	4
2	1

128	64
32	16

▲ Znorlian

In Znorlian, \wedge means “one.”

In Znorlian, the order of digits doesn't matter, so

$$\wedge \bowtie = \bowtie \wedge \quad (\text{Which is five.})$$

What is the shortest way to write 21 in Znorlian?



◀ Znorlian

In Znorlian, there are different ways to write some numbers. For example:

$$\bowtie \bowtie = \blacksquare = \nabla \nabla \nabla \nabla$$

What is the shortest way to write 21 in Znorlian?

○ Znorlian

Here are three ways to write “seven” in Znorlian:

$$\nabla \wedge \nabla \nabla = \nabla \bowtie \wedge = \bowtie \nabla \wedge$$

What is the shortest way to write 21 in Znorlian?

▼ Znorlian

Adding numbers in Znorlian is easy. Just put them together. For example,

$$\wedge \wedge = \nabla$$

is an addition sentence adding \wedge and \wedge .

What is the shortest way to write 21 in Znorlian?

▶ Znorlian

Znorlians write subtraction using a minus sign, just as we do. (Their equals sign is the same, too.) In Znorlian,

$$\bowtie - \nabla = \nabla \blacksquare \bowtie$$

What is the shortest way to write 21 in Znorlian?

△ Znorlian

With the first five digits in the Znorlian number system (not in order):

$$\nabla \bowtie \bowtie \blacksquare \wedge$$

You can write every number from 1 to 31 without using any double Znorlian digits.

What is the shortest way to write 21 in Znorlian?

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▲ Lorith

In Lorith, they write numbers with digits like ours, but the “places” are different. Only the “ones” place is the same.

So while **1** (in Lorith) = 1 (for us),
111 (Lorith) = 9 (us).

Your job, as a group, is to make a Lorithian multiplication table up to 110 by 110.



◀ Lorith

In Lorith, the only digits that can appear in the last place are **0** and **1**. In the next-to-last, **0**, **1**, and **2** are OK.

That means that **12** is not a number for the Lorithians, while **21** is. (It means five.)

Your job, as a group, is to make a Lorithian multiplication table up to **110** by **110**.

○ Lorith

Lorithian mathematicians have long known that multiples of three always end in **00** (as in **200**, which is 12) or **11** (as in **311**, which is 21).

Extension: You can use this fact to check your multiplication table, but can you and your group explain why this is true?

▼ Lorith

In Lorith, the third digit from the end is the “sixes.” That means that

100 (in Lorith) = 6 (for us).

Your job, as a group, is to make a Lorithian multiplication table up to **110** by **110**.

▶ Lorith

You need four digits in Lorith to represent any number larger than 23. For example,

2010 (in Lorith) = 50 (for us)

Your job, as a group, is to make a Lorithian multiplication table up to **110** by **110** (don’t worry—that’s only eight by eight).

△ Lorith

Here are some Lorithian addition facts:

$$\begin{aligned} 21 + 21 &= 120 \\ 100 + 100 &= 200 \\ 101 + 101 &= 210 \\ 110 + 110 &= 220 \end{aligned}$$

Use them to help your group make (or check) a Lorithian multiplication table up to **110** by **110**.