LessonTitle: # Fractal FUNctions **Geo 6.5b**

Utah State Core Standard and Indicators Geometry Standards 1-4 Process Standards 1-5

Summary

In this lesson, students build fractals and track the growth of fractal measurements using tables and equations. Concepts of similarity and area are explored. Also included is research activity involving a presentation on fractals and their history. This research presentation could be used as an introduction to fractals. The Chaos game is also included. Please refer to Geo 6.5 for a lesson on creating fractals using transformations and rotations.

Enduring Understanding

- Most mathematics you study in school is old knowledge. But mathematicians and scientists currently research the geometry of fractals to understand nature's forms and patterns! By studying and creating fractals, students learn about applying mathematics in today's technological world.
- Fractal development is *"Endlessly repeating, Ever diminishing, Self similar."* The iterated fractal development can be studied and communicated using algebraic representation.

Essential Questions

- What are the patterns of fractal development and how can we describe them using Algebra?

Skill Focus

- Function patterns and equations—algebra review
- Scaling
- Self-similarity
- Geometric transformation, complexity, and limits.

Vocabulary Focus

Assessment

- Students create function patterns for their own Koch curves. See the last part of the Fractal Function activity below. They could create the function questions using the three examples below, then find the answers to their own questions, then exchange and evaluate each others work. Use a class developed rubric for evaluation.
- Students develop their own fractal and write the transation codes.

Materials: Isometric and regular dot paper, Calculators, Computers, videotapes ordered from the district

Launch

Explore

- What are the basics of fractals?
- What is iteration and how does it work in creating or studying fractals?
- How do we use geometric transformation to understand or create fractals?

Summarize

Apply

Directions:
Prior to doing this activity, you could have students do "What's My Sum" and "Sum Me Up" activities found on pages 63-64 and 119-120 in NCTM's Navigating Through Geometry Grades 9-12.

Many students and teachers know little about fractals. As in all lessons the discussion or hook activity is important. We suggest the following.

- Activities which give students experience with similarity. Try Geo 7.0, "Growing Generations of Similar Figures."
- Have students build larger and larger self similar figures using linking cubes.
- Show a video clip from Jurassic Park in which they talk about fractals and chaos.
- To introduce the idea of "Endlessly repeating, Ever Diminishing, Self Similar," demonstrate an animated fractal building process. (Can be found in Geometer's Sketchpad version 4, Sketches folder/ Geometry/ Fractal Gallery.)
- Show a video tape ordered from the district media center. Jordan School District presently owns 3 videotapes about fractals.

Have students research the web to answer specific questions about fractals. See the Fractal Research cooperative power point activity below. Show the power point introductory questions. Then let students talk about what they think fractals might be. Then divide into groups for internet research.

Students create fractals on isometric and plain graph paper. Then track the development (growth or shrinking) toward a limit.

Students rotate and reflect to create variations of the Sierpinski Gasket.

Assessment:
Assess student products using predetermined criteria and rubrics. See assessment under teacher info.

Extensions:
Students could learn about patterns in chaos and play the chaos game. See below.

Students could create fractals using Geometer's sketchpad or a drawing program. See below.

Fractal FUNction I. The Sierpinski Gasket

Draw an equilateral triangle with a side length of 27 units on isometric dot paper. This is Stage 0. To build a 3rd stage Sierpinski Triangle, repeat the following 3 times.

Fractal Building Algorithm for the Sierpinski Triangle

- Locate the midpoints of each side of the triangle.
- Connect the midpoints to create four similar triangles within the larger triangle.
- Throw away the center.
- Repeat the process in the remaining triangles

Complete the table below.

Stage	No. of Triangles	No. of Holes	New Triangle Area	Total Area	New Triangle Perimeter	Total Perimeter
0	1	0	1	1	3	3
1	3	1				

Number of Triangles

Predict the number of triangles in a 10^{th} stage Sierpinski Triangle.______ What did you do to find that prediction?

Create a formula for the nth stage. Number of triangles (t) = ________________________

At what stage in the Sierpinski triangle will the number of sub-triangles first be large enough to assign a separate one to each and every person on earth? SHOW your thinking.

Area of Triangles

Predict the remaining area in a 6^{th} stage Sierpinski Triangle._______ What did you do to find that prediction?

Create a formula for the nth stage. Triangle area (a) = ____________________________

As the stage gets larger without bound, what happens to the shaded area?

Questions:

1) If the edge of a stage 0 triangle is 1 foot, will the perimeter ever get large enough to exceed the circumference of the earth? Why or why not?

2) What would remain of the original large triangle after four iterations if the algorithm were changed to keeping only the inner triangle? Explain.

Extra Credit: Develop formulas for Holes, Total Area, Total Perimeter.

Fractal FUNction II. The Square Carpet

Draw a square with a side length of 27 units on regular dot paper. This is Stage 0. To build a 4th stage square carpet, repeat the following algorithm 3 times.

Fractal Building Algorithm for Square Carpet

- Trisect each side of the square.
- Connect the trisection points to create 9 squares within the square.
- Throw away the center.
- Repeat the process in the remaining triangles

Complete the table below.

Stage	No. of Squares	No. of Holes	New Square Area	Total Area	New square Perimeter
0	1	0	1	1	4
1	8	1			

Develop formulas for:

No. of Squares (s) = ____________________ No. of Holes (h) = _____________________

New Square Area (A_n) = ________________ Total Area (A_t) = ______________________

New Square Perimeter (P_n) = _____________________

Questions:

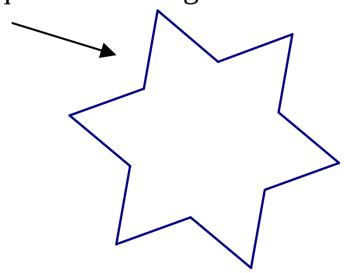
1) If this iterated (repeated) process were to continue without end, what would happen?

2) Suppose the algorithm were changed from keeping the eight border sub-squares to keeping only the four corner sub-squares. What figure would emerge after two iterations?

Fractal FUNction III. The Koch Snowflake

On a piece of isometric dot paper center a drawing of an equilateral triangle that is 27 units on each side. That is stage 0. Stage 1 looks like this.

- Describe the building algorithm



To create a Koch Snowflake, repeat the algorithm 4 times and complete the table below.

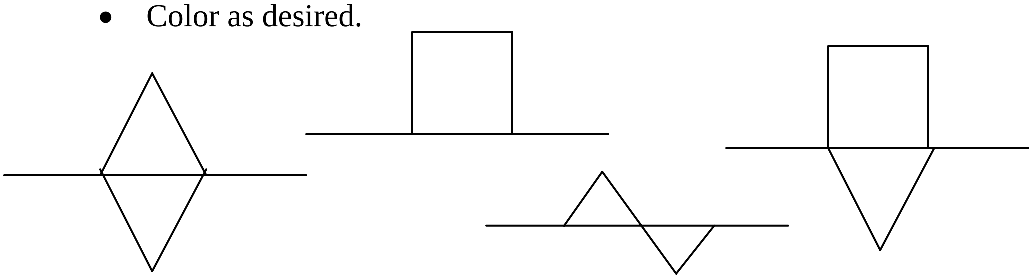
Stage	Total Area (fraction	Total Area (decimal)	Perimeter (fraction)	Perimeter (decimal)
0	1		3	
1				
2				
3				
4				
10				
n				

- When will the snowflake become too large for the paper?
- What happens to the area?
- What happens to the perimeter?
- Describe your ideas about what a “limit” might mean in mathematics.
- Were there any limits in the Sierpinski Gasket or the Square Carpet? If so describe them.

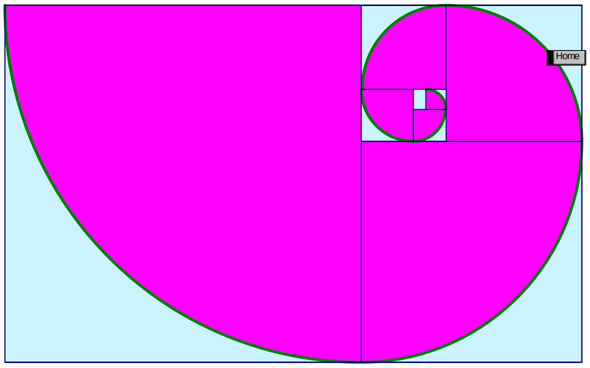
IV. Your OWN Koch-like Curve.

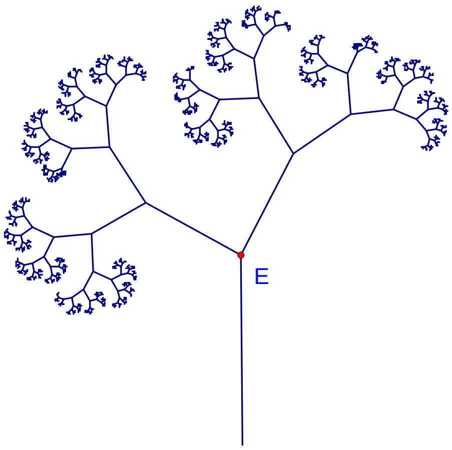
Draw a line on isometric graph paper, the long way—making certain the line can be divided into 3 or 4 equal segments. This is stage 0. Then….

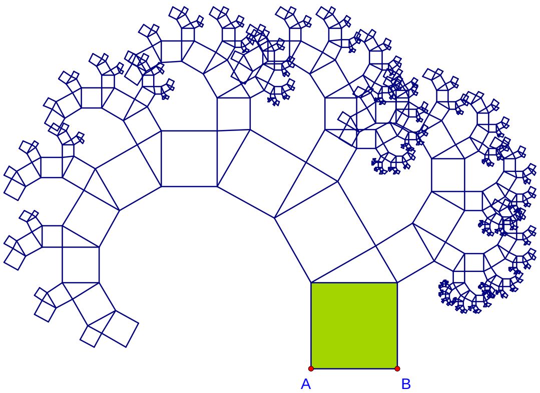
- Divide the line into segments.
- Create your curve pattern.
- Iterate for each new line segment.
- Repeat the iteration until you are satisfied with the end product.
- Color as desired.

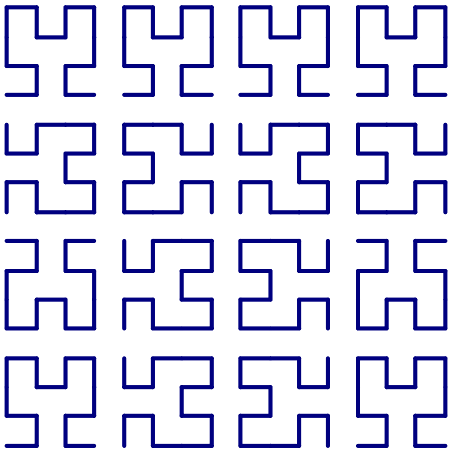


V. Are They Fractals? Why or why not?









The Chaos Game

Play this game and watch the apparent chaotic behavior of a moving point. Begin doing this by hand. Then continue using the program on the calculator.

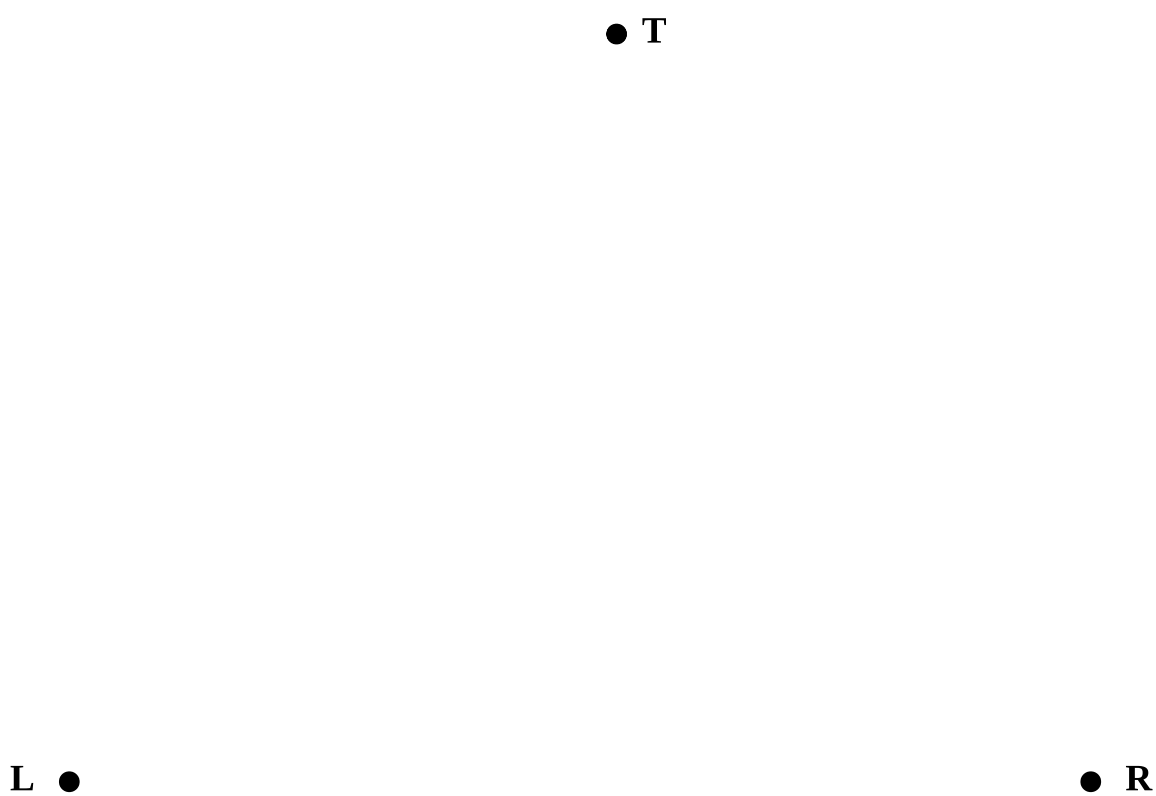
Start with any point inside the triangle formed by vertices L, T, and R.

Step 1 Roll the die and move according to these rules.
- For 1 or 2, move halfway to L.
- For 3 or 4, move halfway to T.
- For 5 or 6, move halfway to R.

Step 2 Connect the point to this newly located midpoint.

Step 3 Starting from the last midpoint located, repeat the steps.

Continue repeating the process extending the path to four successive midpoints.



Fractal Research

A cooperative Power Point Presentation

What's so cool about fractals?
Most mathematics you study in school is old knowledge. Mathematicians and scientists currently research the geometry of fractals to understand nature's forms and patterns!

Research on the internet to find answers to the questions below. Prepare a powerpoint presentation to share with the class.

- Open a new Power Point Presentation.
- Make a title page and include the names of those in your group.
- Access the internet to research information about fractals.
- Create your presentation slides to answer some of the questions below.
- Create one slide to show your favorite fractal from those you observed on the fractal sites.
- Your research presentation slides will be evaluated for quality not quantity.

1. What are fractals?
2. Why study fractals?
3. What are the mathematics of fractals?
4. Where are fractals in the world?
5. What is the history of fractals
6. What is fractal research used for?
7. What are scientists and mathematicians learning from fractals?
8. How are chaos and fractals related?
9. How do you make a fractal?
10. What are some famous fractals and why are they famous?