LessonTitle: Measuring Cones, Pyramids and Spheres **Geo 5.6**

Utah State Core Standard and Indicators Geometry Standards 4.2 Process Standards 1-5

Summary

In this lesson, students first derive the formulas for volume of cones, pyramids and spheres by relating the volume of a cone to a cylinder of the same base and height, a pyramid to a prism of the same base and height, and a sphere to a cube of the same base and height. In a second activity, students find the maximum cone volume using a paper circle with a radius slit.

Enduring Understanding	Essential Questions
Geometry enables us to describe, analyze, and understand our physical world. We derive formulas by analyzing what we know to be true and then generalizing and using the formulas.	How are the volumes of cones pyramids and spheres related to cylinders, prisms, and cubes? Why is volume and surface area important?
Skill Focus	**Vocabulary Focus**
• Deriving formulas and finding areas of cones, pyramids, and spheres and surface area of spheres	

Assessment

Materials: Calculators, measuring tools, rice or sand or other filler material, View Thru Geometric Solids [with congruent heights and bases (if applicable) and radii (if applicable)]or student constructions, see below.

Launch

Explore

- How does the volume of cones and pyramids relate to cylinders and prisms?
- How can we think about the volume of a sphere? Can it be related to the volume of a cylinder or cube?
- How can we think about the surface area of a sphere?
- How do surface areas of different solids with the same base and height compare?
- How do you create a cone with maximum volume?
- For what real world situations would we need to find surface area and volume?

Summarize

Apply

Directions:

For 5.6a, derive the formulas. Practice using them. Students should come up with the following formulas.

- V(cones and prisms) = 1/3 Bh
- V(sphere) = 2/3 $\pi r^2 h$ or 2/3 Bh
  (Because the height is twice the radius, the simplified equation will be $4/3\pi r^3$.)

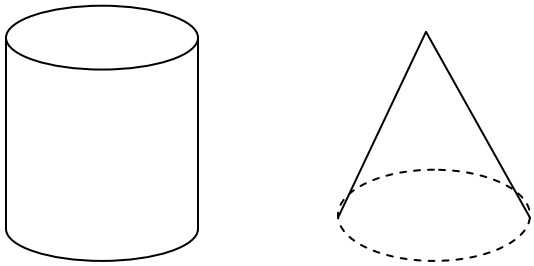
Included below is a question for estimating surface area. It needs to be adjusted in order to become a legitimate task.

For 5.6b, students will build, conjecture and then test their ideas about the maximum cone volume. Use beans to estimate volume. Use stir straws to find the height of the different cones—mark the straw with pen or marker.

Geo 5.6a Volume of Cones, Pyramids and Spheres

1) How does the volume of this cone compare to the volume of the cylinder?

Assume that their bases are congruent and they both have the same height.



If you fill the cone with rice or other filling material, predict how many cones of rice will fit into the cylinder. _____

2) Try it. How many cones fit into the cylinder?_____ About what fraction of the cylinder is filled by the volume of one cone?_____

3) What about a pyramid and a prism with the same bases? Predict how many pyramids filled with rice will fit into a prism. _____

Try it with two or three sets of same-base pyramids and prism. How many pyramids fit into same-base prisms?_____ About what fraction of the prism is filled by the volume of one pyramid? _____

4) Compare your results with the results of others. Did you get similar results with both your pyramid-prism pair and the cone-cylinder pair? You should be ready to make a conjecture

Formula for finding the volume of a pyramid or cone:

V (pyramid or cone) = _____

Explain your formula.

5) How can we think about the sphere's volume of a sphere? Let's examine how the volumes of spheres, cones and cylinders (with the same height and radius) compare?

Predict how many sphere volumes will fit into a cone or a cylinder? _____

Predict how many cone volumes will fit into a sphere?_____

Try combining the volume of the sphere and the cone into the cylinder. What happens?

_____

About what fraction of the cylinder is the sphere? _____

You should be ready to make a conjecture

Formula for finding the volume of a sphere: V(sphere) = _____

Explain your formula. Simplify the formula if possible.

Estimating Surface Areas of Solids

How can we turn the following question into a good one?

Here is a list of fourteen solids with the same heights and bases (if applicable) or radii (if applicable). Order these solids by surface area, from large to small.

1.____ 2.____ 3.____ 4.____ 5.____ 6.____ 7.____ 8.____ 8.____ 9.____

10.____ 11.____ 12.____ 13.____ 14.____

a. large cube
b. small cube
c. hemisphere
d. sphere
e. large rectangular prism
f. small rectangular prism
g. large triangular prism
h. small triangular prism
i. pentagonal prism
j. triangular pyramid
k. square pyramid
l. large cylinder
m. small cylinder
n. cone

Compare your results with others in the class. Prove your answers.

- SA (prisms, cylinders, pyramids) = surface area of all lateral surfaces added together
- SA (sphere)= $4\pi r^2$
- SA (cone) = $\pi r l + \pi r^2$ (l= slant height of cone)

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