LessonTitle: **Squares and Square Roots** **Alg 8.5**

Utah State Core Standard and Indicators Algebra Standards 1.3.1, 2.2.5 Process Standards 1-5

Summary

In this lesson, students draw squares on dot paper—some are square with the paper and some are on a diagonal. They build a table comparing the areas of the squares to the side length. They must use square root notation to describe the length of the sides of the squares drawn on the diagonal. Then students find the radical numbers to describe the length of the sides of squares whose areas are known. They can use the Pythagorean theorem to check their answers. **Please Note! There are three versions of this activity below. The first version is very open and requires the students to investigate and draw conclusions. The second and third versions lead students todifferent specific learning in the investigation process.** Also, please be certain to access the "Writing About Squares" independent investigation below.

Enduring Understanding	Essential Questions
The length of an edge of a square can always be found by finding the square root of the area of a square.	How do square roots relate to squares? How do we investigate patterns?

Skill Focus	Vocabulary Focus
• Squares and square roots • Radical expressions • Ordering radical numbers • Review ordered pairs and equations of lines	

Assessment

See below

Materials: five-dot by five-dot grid worksheet (see below), You may wish to access the sketch "squares" found in "fundamentals" found in the ready-made sketches from Discovering Algebra with Geometer's Sketchpad.

Launch

Starter: Have students explain in writing the formula A = length x width. They should include a drawing with their explanation.

Explore

- Have students draw all the squares they can on the grid sheet. (included below) Then they can complete the worksheet.
- Use your preferred version of the activity below.

Summarize

Apply

Additional Information:

- You may wish to access the sketch "squares" found in "fundamentals" found in the ready-made sketches from Discovering Algebra with Geometer's Sketchpad. You could refer to and use the "Squares and Square Roots" activity found in Exploring Algebra with Geometer's Sketchpad pages 31-32.
- For practice ordering radical numbers with other real numbers, access Alg 6.6 A question of Order.
- **A great followup to this lesson can be found at** http://math.rice.edu/~lanius/Geom/irrat.html

Continue below

Sample solutions to the "Writing About Squares" investigation below:

"Students should notice that squares with the same slopes have patterns when it comes to area. A few examples:

1. Slopes of zero and undefined (unslanted squares) have areas that are perfect squares.
2. Slopes of 1 and -1 have side lengths with multiples of the square root of 2. The areas can be written in the formula $2n^2$, where n is the number of diagonal units on one side of the square, or $\sqrt{2}$ units.
3. Interesting pattern: Using the slopes ½, 1/3, ¼, etc. and drawing the smallest possible squares will create areas equal to the perfect squares + 1.

"(For the slanted squares) Using the Pythagorean connection, students should eventually realize that the slopes of the sides must be a ratio of two whole numbers. That means that the length of each side of each possible square must be the sum of two perfect squares (ofr for unslanted, one perfect square). For example

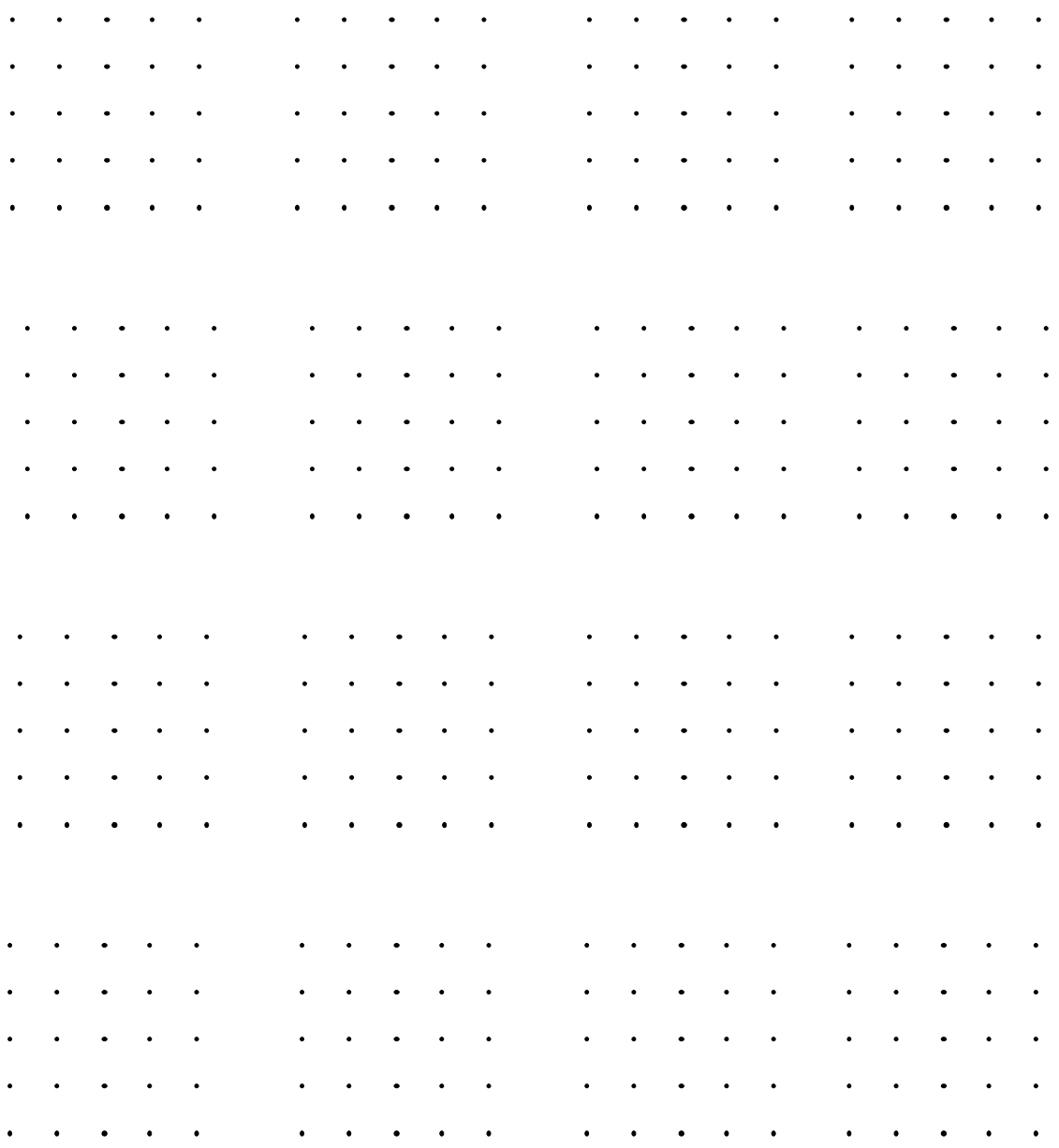
Possible square areas	As a sum of one or two squares
1	1^2
2	$1^2 + 1^2$
4	2^2
5	$1^2 + 2^2$
8	$2^2 + 2^2$
9	3^2
10	$1^2 + 3^2$

"Obviously, the sum of squares is just $a^2 + b^2$ where a and b are the rise and run of the slope of the side. So the rule is that each possible area can be written as a sum $a^2 + b^2$, where a and b are whole numbers. It may be helpful to create a chart of all possible square areas where the numbers a and b are on the axes, and the intersections have the value of $a^2 + b^2$ (set up like a multiplication tables chart). "

"After students answer question b, they will have a list of many possible areas, and might start looking for a pattern of howmany areas are contained in an n by n dot grid. I found it works better to encourage them to find a rule for WHICH areas they can find. Trying to find a pattern gets tricky when they realize that (for instance) a 7 by 7 dot grid has a maximum area of 36, yet it is possible to create a square with area 34, but not until you have a 9 by 9 dot grid. Why? Because $34 = 3^2 + 5^2$, and there is not room to draw a square with that slope on a 7 by 7 (try it!). To have enough room to draw a square with slope a/b on an n by n dot grid (a and b are unit lengths, not # of dots), the rule is that a + b is less than n – 1. There are n dots on each side, and you need room for both a and b on each side of the square, and n dots = n – 1 unit lengths."

"Note: There are many (MANY) more patterns and ideas that you could come up with. I didn't list them all."

Five-dot by Five-dot grids



Alg 8.5 (version one) Squares and Square Roots

1) Draw all the different squares you can on five-dot by five-dot grids. Use the grid worksheet.

- How many different areas can you find?

- What observations can you make? Did you see any patterns? Did you notice anything about the slopes of the lines?

2) Where do you think the term ‘square root’ comes from?

Alg 8.5 (version two) Squares and Square Roots

1) Build as many squares as you can on 5 by 5 pieces of centimeter dot paper. (They do not have to be square with the edges of the paper—they can be drawn diagonally but still be squares.) Each member of your group can draw different ones. If you need more paper, that's fine.

- Find the area and the length of the side. (No rulers)
- Label the squares with the side lengths and the areas.
- Record the squares below.
- You should not measure until you have estimated!

2) Show how you arrived at the lengths of the sides when you couldn't count centimeters?

3) Compare your approximated length of a side to the measured length of a side.

4) What is the edge of a square with areas of
36 _____, 100_____, 144_____, 400_____,

How do you know?

Square	Area	Estimated Side Length	Measured Side Length
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
16			

5) Estimate the edge of squares with areas 17, 57, 95. Then use the square root key to see how close you were.

	17 cm^2	57 cm^2	95 cm^2	115^2		
Estimate						
Calculated						

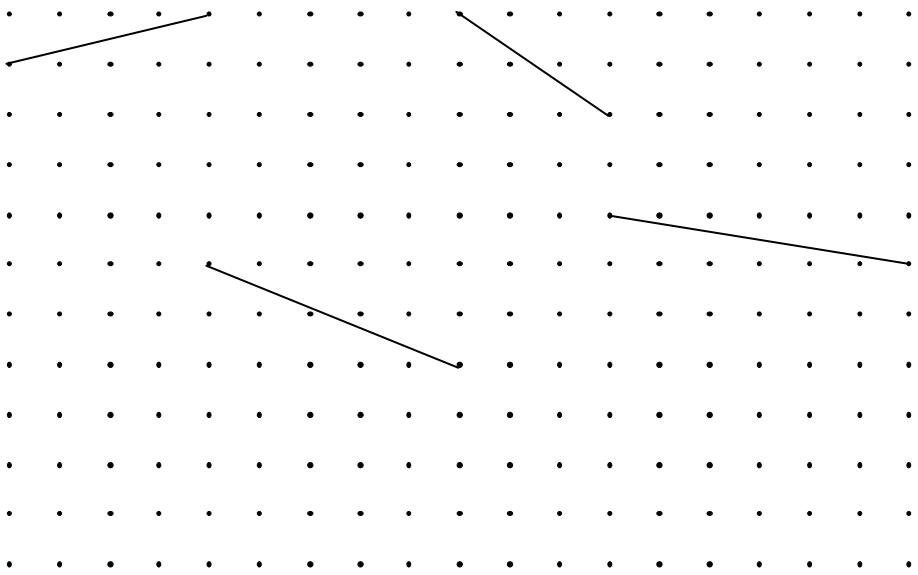
6) Order the following numbers from least to greatest. (Estimate—don't use the square root key.

$\sqrt{8}$ 2.7 $\sqrt{3}$ 1.7 1.3 $\sqrt{5}$ $\sqrt{2}*\sqrt{2}$

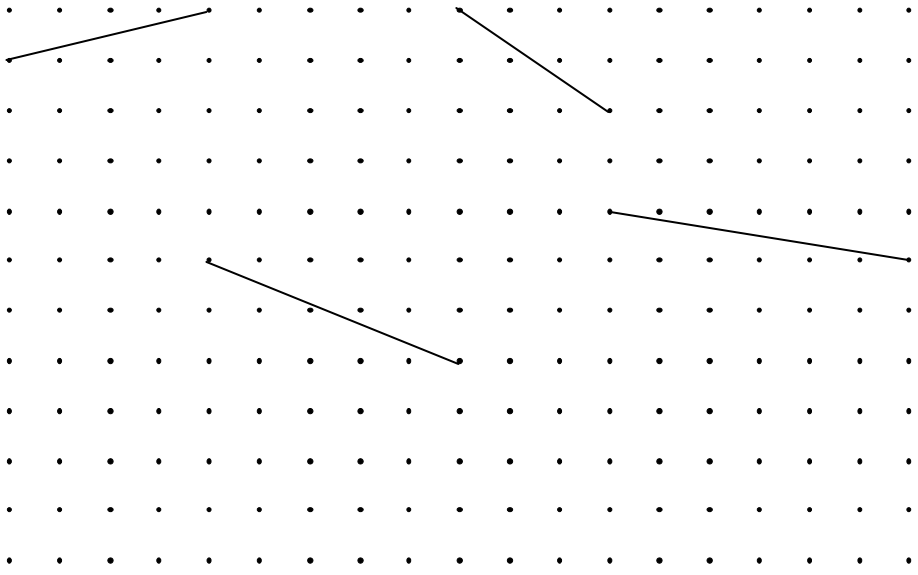
7) Where do you think the term 'square root' comes from?

8) Find the length of the following line segments. No rulers allowed!

Show all your work below or at the side.



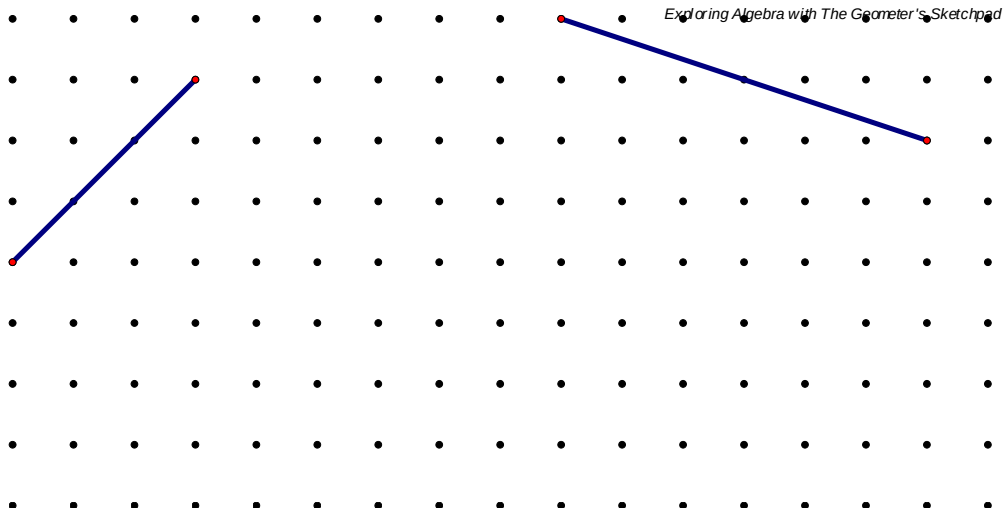
9) Extra for experts: Find the length of the line segments using a different strategy. Show all work.
(Hint: Remember the Pythagorean Theorem.)

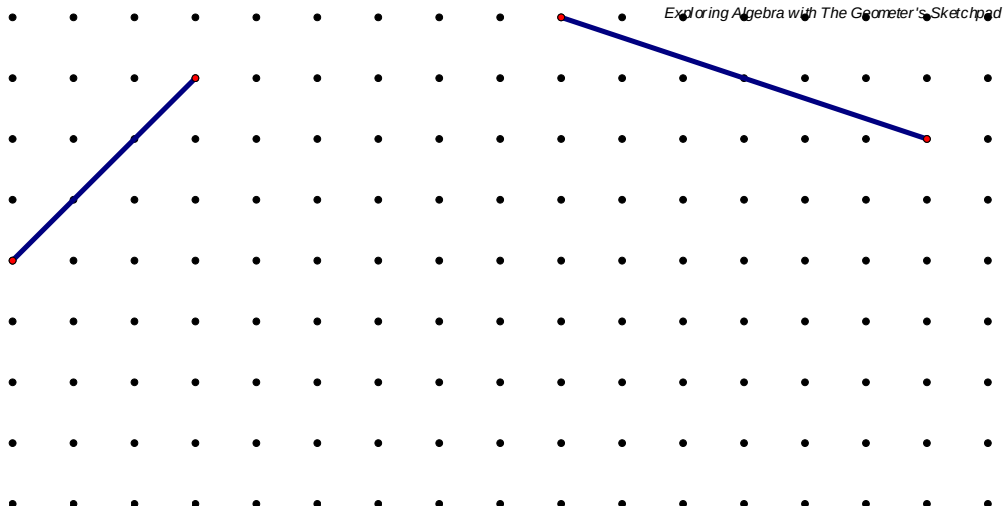


Alg 7.5b Squares and Square Roots Assessment

Calculate the length of the line segments below. No rulers allowed.

Show and explain your methods below or at the side.

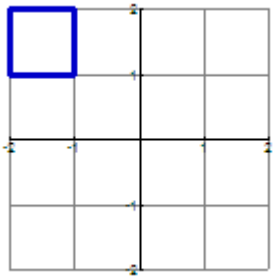
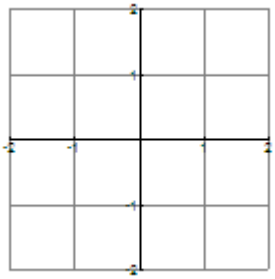
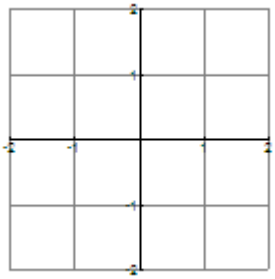
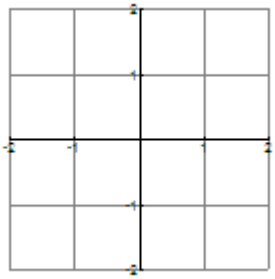
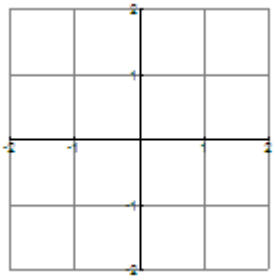


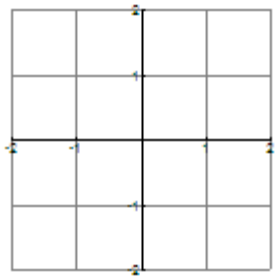
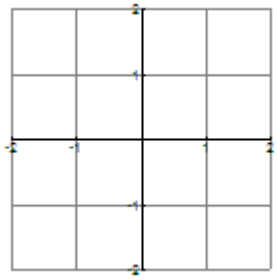
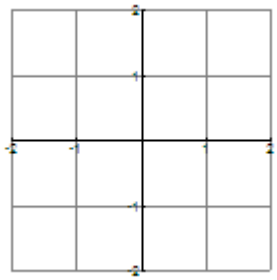


=

Alg 8.5 (version three) Squares and Square Roots

1. On the coordinate planes below, draw 8 squares, **each having a different area**, filling in the chart for each square that you draw.

	Vertices		Equation of the line of each side (in slope-intercept form)	
#1	(-2, 2)	(-1, 2)	y = 2	
	(-2, 1)	(-1, 1)	y = 1	
	Area = 1		x = -2	
	Side length = _____		x = -1	
#2	Vertices		Equation of the line of each side (in slope-intercept form)	
	Area = _____			
	Side length = _____			
#3	Vertices		Equation of the line of each side (in slope-intercept form)	
	Area = _____			
	Side length = _____			
#4	Vertices		Equation of the line of each side (in slope-intercept form)	
	Area= _____			
	Side length = _____			
#5	Vertices		Equation of the line of each side (in slope-intercept form)	
	Area = _____			
	Side length = _____			

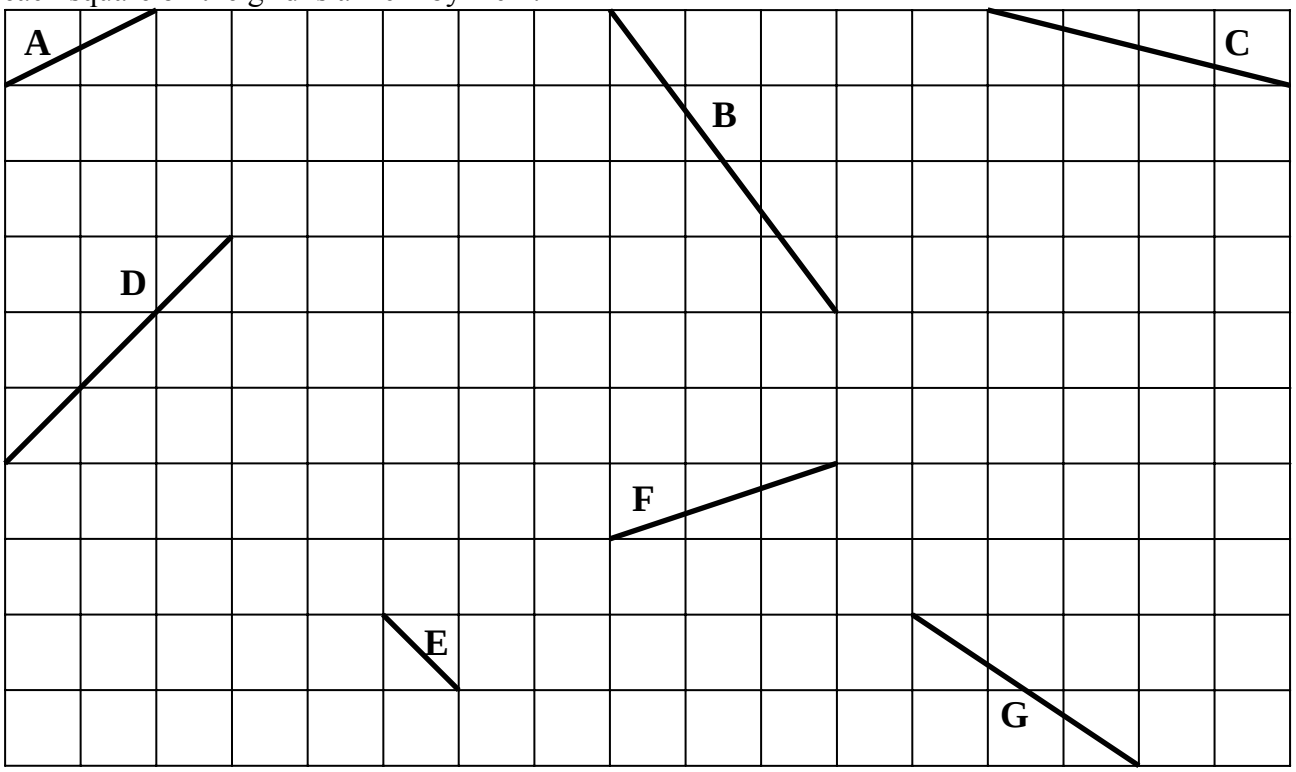
	Vertices		Equation of the line of each side (in slope-intercept form)	
#6				
	Area = _____			
	Side length = _____			
#7	Vertices		Equation of the line of each side (in slope-intercept form)	
	Area = _____			
	Side length = _____			
#8	Vertices		Equation of the line of each side (in slope-intercept form)	
	Area = _____			
	Side length = _____			

2. Where do you think the term ‘square root’ comes from?

3. For the squares with areas of 2, 5, 8, and 10, explain the relationship between the slopes of the lines of each side.

4. Explain, in detail, how you discovered that one of the squares had an area of 10.

Find the slope of each line and estimate the length of each of the following line segments by measuring them with the centimeter side of a ruler. Then, draw a square with the segment as one of the sides and express the length of the segment using the √ symbol. Make sure none of the squares overlap. Note that each square on the grid is a 1 cm by 1 cm.



	A	B	C	D	E	F	G
Slope	$\frac{1}{2}$						
Length (approximate)	2.2 cm						
Length (exact)	$\sqrt{5}$						

5. Do you think that is it possible to find the square root of any whole number using the grid and the method above? Why or why not?

6. Without using a calculator, estimate the values of the following square roots. Be prepared to explain how you came up with your estimate.

	$\sqrt{13}$	$\sqrt{7}$	$\sqrt{11}$	$\sqrt{19}$	$\sqrt{27}$	$\sqrt{26}$
estimate						

Writing about "Squares"

An Independent Investigation

You have investigated the areas of squares with different sloped sides. Now it's your turn to investigate.

- Investigate the questions below.
- Write a short paper explaining your investigation and conclusions. This paper can be typed or neatly written and free from obvious grammatical and punctuation errors
- Use diagrams, tables, equations and graphs if they are helpful to explain your reasoning.
- Be creative in trying to find a unique pattern or idea.
- The paper will be evaluated on 1) the ideas and content, and 2) the organization and presentation.
- Due date is _____

1) Is there a pattern in the areas of squares that have the same slope, but different side lengths? Choose three different slopes—search for patterns.
2) If the vertices of the squares fall on the dots, is there a rule for which (different) areas we find? List some areas that we can find on the dot grids. Why can we find an area of 2, but not 3? Why can we find an area of 4 and 5, but not 6?
3) How many total different areas can be constructed on different sized square grids? That is, how many different areas can we find on a 2 by 2 dot grid? A 3 by 3? etc. Can you come up with a rule for which (or how many) areas can be found on an n by n square?
4) A question of your own choosing.

Assessment rubric

Ideas/Content	Organization/Presentation
5 Student got the main ideas and more, has interesting details, examples, explanations, no filler, and answered the questions.	5 The paper has an introduction and conclusion. The information is in a logical order. The diagrams, tables, etc. help the reader understand the information.
3 Student answered the questions, perhaps not completely. Some details and explanation missing.	3 The paper is missing an introduction or conclusion. The diagrams are not as helpful as they should be. Some confusion.
1 Little or no explanation or original ideas, very few examples or details.	1 The paper is missing an introduction and conclusion. The reader cannot understand the ideas presented. Information is not in a logical order.