LessonTitle: **Function Projects**

Utah State Core Standard and Indicators Algebra Standards Process Standards 1-5

Summary

Students have already learned about exponential and quadratic growth in earlier activities. These projects are an opportunity to reinforce that learning and investigate and present group projects. Students will select an experiment, collect and analyze data, and make presentations about their findings.

Enduring Understanding

Nonlinear functions are found in many real world situations. Growth patterns and rates of change determine the equations and graphs of nonlinear functions.

Essential Questions

- What are some contexts in which we find exponential and quadratic growth?
- How do you identify exponential and quadratic growth?
- Does exponential change graph continue forever? Does it approach a limit?

Skill Focus

- Exponential and quadratic functions and graphs
- Rates of change
- Limits
- Using graphing calculators.

Vocabulary Focus

Assessment

Materials: Graphing Calculators, materials for experiments, student record sheets for all students, Experiment descriptions for each group.

Launch

Explore

Summarize

Apply

Directions:
Refer to previous activities, Choose a Prize (Alg 3.1), Families of Equations (Alg 3.0).

Students have already learned about exponential and quadratic growth in earlier activities. These projects are an opportunity to reinforce that learning and investigate and present group projects.

Involve the class in the preparation of a group performance evaluation rubric (see form below). Some criteria possibilities are: 1) Group understanding of concepts, 2) Clarity of group directions and explanations, 3) Retaining class attention, 4) Questioning techniques, 5) Class learning resulting from group presentation 6) Connections and conclusions, 7) Problem difficulty

First, student groups will select an experiment from those described below. They will

1) collect data, record into lists, create a data plot
2) find the growth pattern and rate of change
3) fit the rate of change into exponential or quadratic equation forms (they may find the regression using the graphing calculators)
4) match the equation graph with the data plot
5) prepare basic problems involving the equation for classmates to solve.

Second, student groups will prepare presentations about their experiment. In the presentation, student groups will…

1) lead the class in the recording of data into lists and creating a data plot,
2) question the class to guide them to develop an equation and create the graph—being the teacher. (Students will complete worksheets for each presentation—see below.)
3) draw conclusions and make connections
4) demonstrate what happens to the graph if the variables change.
5) assist classmates in solving problems. (Students in the class will record information about each experiment on the student record sheets found below.

Note: Investigations involving population growth are more difficult. Groups should understand from the outset that the degree of difficulty will factor into their final point count.
Note: In order to determine equations for population growth problems, groups may need guidance to establish the population base, and the average rate of growth. Once they realize that the population is multiplied by a growth factor, they should be able to come up with an equation where $y = ab^x$ or $\underline{\text{total population}} = (\underline{\text{original population}})(\underline{\text{growth rate}})^{\text{x number of stages}}$

Exponential Function Investigations

Student Record Sheets

TITLE:_____ **Group** _____

Data: **Graph:**

Equations:_____

Observations & Conclusions:_____

TITLE:_____ **Group** _____

Data: **Graph:**

Equations:_____

Observations & Conclusions:_____

Exponential Function Investigations

Directions, Assessment and Experiments

Select an experiment.

1) Collect data, record into lists, create a data plot
2) Find the growth pattern and rate of change
3) Fit the rate of change into the standard form for an exponential equation
4) Match the exponential equation graph with the data plot
5) Prepare basic problems for classmates to solve.

Prepare experiment presentations.

1) Lead the class in the recording of data into lists and creating a data plot,
2) Question the class to guide them to develop an equation and create the graphs—You are the teacher! (Students will complete worksheets for each presentation.)
3) Draw conclusions and make connections
4) Demonstrate what happens to the graph if a, b, or x change.
5) Assist classmates in solving problems. (Students in the class will record information about each experiment on the student record sheets found below.

Evaluation Criteria—how you will be graded: (to be decided upon as a class)

Evaluation Criteria	Points	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8

Exponential Growth and Decay Problems

Rumors: In West Jordan Middle School, Sydney, an 8th grader, decides to start a rumor that Jordan School district is going to declare March 17 a holiday and close school for the day. On the first day of school, she tells 3 students the rumor and gives them instructions to repeat the rumor (and instructions) to 3 more students the next day, etc.

- If each student follows these instructions, how many students will hear the rumor on day 6? By day 6?
- How many students (including Sydney) have heard the rumor?
- If Sydney starts the rumor on March 11 and there are 1,250 students in the school, when will they all know the rumor? (assuming each person tells two new people)
- Describe the growth if every person told were to tell 3 new people every day?

Cookie Monster and Carbon 14: Cookie Monster begins with a whole cookie. Every second he eats half the cookie he has. The following sequence describes this process.

Sec.	Remaining cookie
0	1
1	1/2
2	1/4
3	1/8

Graph the pattern and find an equation to describe the decay.

Some carbon atoms are radioactive. The radioactive carbon atom, carbon-14, has a half-life of about 5700 years. The half-life is the time it takes for one-half the radioactive atoms to decay.

Use the equation from "Cookie Monster" to determine how long it will take for the 50 grams of carbon 14 in a dead tree to completely decay (close to zero grams left).

$y = 50(1/2)^{x}$

Cell Division: At the beginning of an experiment, a laboratory culture dish contains 500 bacteria. The number of bacteria increases by 50% each hour.

- Determine the approximate number of bacteria in the culture dish after three and a half hours, after 1 day, after 3 days.
- Suppose the volume of 500 bacteria is 1 cubic centimeter. When would the bacteria fill the_____(to be decided upon by the class—waste basket, closet, desk, classroom etc).

Skittles.

Part I. Count out 100 Skittles and place them flat in a box top. Overturn the box top into another box top. Remove the ones that land "S" side up. Count how many remain and record that number in the table. (DO NOT EAT YET)

Repeat until all the Skittles are gone, keeping track of the number of Skittles at each stage. At toss 0 there are 100 Skittles. At toss 1 there are _____, etc.

Part II. Take 2 Skittles and overturn two of them onto a box top. For every "s" that is showing add two Skittles to the group. Count the total number of Skittles you have and record the data in a table. Continue until you have no more Skittles to add.

World Population Growth. Based on an estimate that there are 10 billion acres of arable land on the earth and each acre can produce enough food to feed 4 people, some demographers believe that the earth can support a population of no more than 40 billion people.

Suppose the population of the earth continues to grow as it has in the past. How long will it take to reach a population of 40 billion?

1970	3.72 billion
1980	4.47 billion
1990	5.33 billion

Another Credit Card or Investment Investigation (you choose)

Port Disney is a town California. The school board of Port Disney thinks the best kind of elementary schools are small, neighborhood schools. So their schools have about 200 students each. About one-third of the population is of elementary school age. Port Disney's population has been increasing over the past twenty years and the growth trend is expected to continue. How many new schools will Port Disney need and when should they build them?

Year	Population
1970	24,567
1980	31,200
1990	39,312

1) Make predictions for the population of Port Disney for the next 30 years, based on data from past years. Show all work, graphs, etc. to support your predictions.
2) Make recommendations for how many schools will be needed and when the schools should be built. Prepare your report to the school board, complete with graphs.

Aids: The Preventable Epidemic. A recent congressional study reported that AIDS and HIV infections among teenagers rose an alarming 70% between 1990 and 1992.

Part I. Simulate the spread of an epidemic. Place a cup of red beans (healthy people) and one white bean (an infected person) in a large flat containers such as a cafeteria try or a cardboard box lid. Shake the box and count how many red beans are touching or within 1 mm (a dime's thickness) of the white bean. These beans are infected with the virus. Replace the infected red beans with white beans. Shake again. Continue the pattern and keep track of the number of white beans at each shake. At 0 shakes, there is 1 white bean. After 1 shake, record the white beans, etc.

Part II. The following data show the total US AIDS cases reported for people ages 13-29 from 1980 to 1991. Compare the simulation from part I with the real data graphs and equations.

Year	No. of Cases	Year	No. of Cases
1980	18	1986	7897
1981	79	1987	13,307
1982	301	1988	19,998
1983	886	1989	27,999
1984	2064	1990	37,022
1985	4296	1991	48,007

Drugs, a Diminishing Return. You are an Olympic athlete scheduled to compete on Friday at 4:00 PM. On Thursday morning you awaken with a bad cold and consider taking a cold medication. You know drug testing will take place immediately prior to the competition and the drug test is capable of detecting 1 or more milligrams of medication in your system. After each 4-hour time period your body will have dissipated one-fourth of the remaining drug. If you take a 16 milligram dosage at 8:00 AM, will you pass the drug test and be able to participate?

Simulation:
Step 1: Fill one container with four cups water to which you add 16 mg.(drops) of red food coloring (cold medication). Have a second container of clear water available.

Step 2: Pour out 1 cup of solution and replace with 1 cup of pure water. (dissipation of one-fourth of the drug in a 4 hour period).

Step 3: Repeat step 2 three more times.

Step 4: Follow the prescribed assignment sheet to create lists, graphs, an equation. Use 4 hour periods as the x value and milligrams of drug remaining as the y value.

Hot Chocolate: How fast does hot chocolate cool down?
Method 1: Pour boiling water into different cups. Use a glass cup, a Styrofoam cup, an insulated mug etc. Record the temperatures and the times. Repeat the measurements every minute for a given amount of time.

Method 2: Use CBLs and temperature probes to collect data about the cooling of boiling water poured into different cups.

Follow the prescribed assignment sheet to create lists, graphs, and an equation. Use 4 hour periods as the x value and milligrams of drug remaining as the y value.

Population Decline: The population of Funtown is declining. If the population continues to follow the pattern of decline in the following data, predict the population in 2005, 2020?

1985	100,000
1990	90,000
1995	81,000
2000	72,900

Follow the prescribed assignment sheet to create lists, graphs, and an equation. Use 4 hour periods as the x value and milligrams of drug remaining as the y value.

Fibonacci's Rabbits
For this investigation you must know that rabbits can have a set of babies the second month a males and female are together. They can have a new set of babies every other month thereafter. Say that each time a pair of rabbits has babies, they have exactly 1 boy and 1 girl.

Your investigation will be to see how the total number of pairs of rabbits grows each month. Consider the first month as stage 1 with one pair of rabbits. The second month (stage 2) also has only one pair of rabbits.

A Drop in the Bucket Pretend that you are painting cubes of different sizes. Consider a single 1x1 unit cube as a stage 1 cube. So a 2x2 cube would be a stage 2 cube, made up of 4 unit cubes stuck together. If you drop the cubes into a bucket of bright green paint, and then take them apart, how many of the cubes in each stage are painted on three faces, two faces, one face, and no faces?

Make a table, graph and equations to describe the patterns you discover. (See handout)

Towers of Hanoi: The game consists of three pegs and increasingly large donut type disks. The objective is to move the disks one at a time to another peg in the fewest possible moves. A larger disk cannot be placed on a smaller disk. And the disks must be arranged from largest on the bottom to smallest on the top.

Disks Moves Pattern Make a table, graph and equation.

This game is called the Tower of Hanoi. The original tower is in the great temple of Benares, India. The story claims that there are three diamond needles and sixty-four golden disks, graduated in size near the temple. The monks from the temple move the disks. When all disks have been transferred, the world will come to an end. If the monks started in year 0, in what year will the world come to an end?

The Handshake Problem Make the table and graph and find the equation for the growth of the number of handshakes when each person shakes hands with the others in the room once and only once.

The Price and Size of Pizza

A price list for pizza usually shows the prices for pizzas of different diameters. The price increases as the diameter increases. You will examine a price list for several different size pizzas with the same ingredients to determine the relationship of the price to the diameter.

1) Make a table with columns for diameter, area, and price. Record the data.

2) Use your data to make a graph. The x axis = diameter. The y axis = price.

3) Make a second graph using x axis = diameter and y axis = area.

4) Compare the graphs. Summarize and explain the graphs