

LessonTitle: Linear, Exponential and Quadratic Patterns **Alg 7.1**

Utah State Core Standard and Indicators Algebra Standard 2.1 Process Standards 1-5

Summary

In this lesson, students examine the patterns in nonlinear data. They relate these patterns to the standard exponential and quadratic functions.

Enduring Understanding

Relationships found in physics, geometry and other real-world situations often can be expressed as rules in the form of equations. By studying the patterns of change in data, we can identify the patterns which produce linear, exponential and quadratic equations

Essential Questions

What are the patterns of change found in real world data?
How do we represent these patterns using mathematics?

Skill Focus

- Patterns and functions
- Problem solving

Vocabulary Focus

Assessment

Materials

Launch

Explore

Summarize

Apply

Directions:

Refer to activity 5.0, families of equations. If students still have the activity refer to it. If not, make 1 copy per group (included below).

Ask students to examine the tables to see if they can identify from the data which functions are linear, exponential, and quadratic.

Lead students to examine the differences in the y values at each stage.

Information about finite difference patterns:

- If the finite difference is constant, then the change is a repeated constant and the equation and graph are linear
- If the finite difference is a multiplies by the same factor every time, then the equation and graph are exponential
- If the finite difference is increases or decreases following a pattern, then the equation and graph are quadratic.

Alg 7.1

Discovering Function Rules

Revisit the Families of Equations assignment from Module 5. Examine the finite differences between y values at each stage.

What do you notice about the finite differences for linear equations? _____

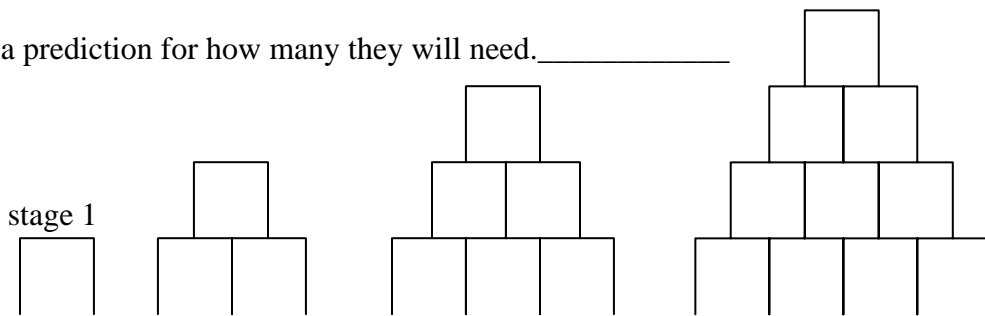
What is the trick for using finite differences to identify exponential equations? _____

The following activity will assist you in discovering how to use finite differences to find quadratic equations.

The Domino House Problem

This domino house has four stories and contains 24 dominoes. Jim and Theresa try to beat the world record of 73 stories for a domino house. Before they begin, they want to know how many dominoes they'll need.

Make a prediction for how many they will need. _____



Use the following table to help you discover an equation.

| x | y | 1 st difference | Pattern in y |
|-----|-----|----------------------------|----------------|
| 1 | 1 | | |
| 2 | 3 | | |
| 3 | 6 | | |
| 4 | 10 | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 73 | | | |

Linear, Exponential and Quadratic Patterns

Finding equations from sequence patterns

- 1) Determine the next 2 stages in the following patterns. 2) Find the sequence rule.
 3) Predict the 10th number. 4) Find the equation for determining the y value at any stage.

To figure out a way to get the 10th number without adding or multiplying repeatedly, find an equation for finding y. In other words, find an equation to tell you what do you do to x to get y at any stage?

Watch for patterns. Find the families of equations. Be prepared to write your thinking.

1)

| x | y |
|-------------|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 10 | — |
| $y =$ _____ | |

2)

| x | y |
|-------------|----|
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 10 | — |
| $y =$ _____ | |

3)

| x | y |
|-------------|----|
| 2 | 6 |
| 4 | 10 |
| 6 | 14 |
| 10 | — |
| $y =$ _____ | |

4)

| x | y |
|-------------|----|
| 3 | 11 |
| 4 | 14 |
| 5 | 17 |
| 10 | — |
| $y =$ _____ | |

5)

| x | y |
|-------------|----|
| 5 | 8 |
| 6 | 10 |
| 7 | 12 |
| 10 | — |
| $y =$ _____ | |

6)

| x | y |
|-------------|----|
| 3 | 5 |
| 5 | 9 |
| 9 | 17 |
| 10 | — |
| $y =$ _____ | |

7)

| x | y |
|-------------|----|
| 7 | 12 |
| 9 | 14 |
| 13 | 16 |
| 10 | — |
| $y =$ _____ | |

8)

| x | y |
|-------------|-----|
| 5 | 13 |
| 10 | 23 |
| 20 | 43 |
| 100 | 203 |
| 10 | — |
| $y =$ _____ | |

9)

| x | y |
|-------------|---|
| 4 | 1 |
| 8 | 2 |
| 12 | 3 |
| 10 | — |
| $y =$ _____ | |

10)

| x | y |
|----|---|
| 6 | 3 |
| 10 | 5 |

11)

| x | y |
|---|---|
| 2 | 4 |
| 3 | 9 |
| 4 | 4 |

12)

| x | y |
|---|---|
| 1 | 1 |
| 2 | 4 |

stage 1 stage 2 stage 3

equation: $y =$ _____ equation family _____

What will the graph be like? _____

2) **Count the edges of squares above.**

(where 2 squares meet is 1 edge)

Sequence Rule: 10th number

equation: $y =$ _____

equation family _____

What will the graph be like? _____

| stage (x) | total edges (y) |
|-----------|-----------------|
| | |

3) **Count the triangles.**

Sequence Rule: 10th number



equation: $y =$ _____

equation family _____

What will the graph be like? _____

| stage (x) | total triangles (y) |
|-----------|---------------------|
| | |

4) **Count the edges of the triangles.**

Sequence Rule: 10th number



equation: $y =$ _____

equation family _____

What will the graph be like? _____

| stage (x) | total (y) |
|-----------|-----------|
| | |


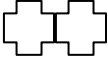

5) **Paper Tearing:** Begin with 1 piece of paper at stage 1. For stage 2, tear it in 2 pieces. For each succeeding stage tear each piece of paper into two. Keep track of the total pieces of paper.

| | | | |
|----------------|-------------------------|-----------|------------------|
| Sequence Rule: | 10 th number | Stage (x) | total pieces (y) |
| | | 1 | 1 |
| | | 2 | 2 |

equation: $y =$ _____ equation family _____

What will the graph be like? _____

6) **Count the crosses.**

| | | | |
|--|--|-----------|------------------|
| Sequence Rule: | 10 th number | stage (x) | total pieces (y) |
| | | 1 | 1 |
|  |  | | |
|  | | | |

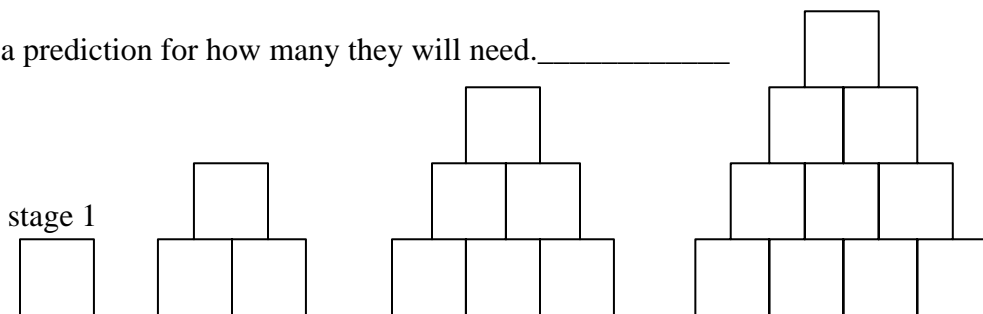
equation: $y =$ _____ equation family _____

What will the graph be like? _____

Formulas for the problems below will be extra credit.

7) **The Domino House Problem:** This domino house has four stories and contains 24 dominoes. Jim and Theresa try to beat the world record of 73 stories for a domino house. Before they begin, they want to know how many dominoes they'll need.

Make a prediction for how many they will need. _____



| | | |
|-----------------------|-----------|-----------|
| equation: $y =$ _____ | stage (x) | total (y) |
|-----------------------|-----------|-----------|

equation family _____

What will the graph be like? _____

- 8) **Gossip:** On day 1, one student tells three other students a secret. On day 2, these three students each tell three more students. Keep track of the new people who know the secret each day as well as the total who know.

| Sequence Rule: | 10 th number | day (x) | new (y ₁) | total (y ₂) |
|----------------|-------------------------|---------|-----------------------|-------------------------|
| _____ | _____ | 1 | 3 | 4 |
| 1 4 | _____ | 2 | 9 | 13 |
| _____ | _____ | 3 | | |

equation: $y =$ _____ equation family _____

What will the graphs be like? _____

- 9) **Fibonacci rabbits:** (Babies come in pairs, It takes 2 months before new pairs can have babies. But after that, rabbits can have a pair of babies every month.)

1 mo, 1 pr 2 mo, 1 pr 3 mo, _____

| Sequence Rule: | 10 th month number | stage (x) | total rabbits (y) |
|----------------|-------------------------------|-----------|-------------------|
| _____ | _____ | | |

equation: $y =$ _____ equation family _____

What will the graph be like? _____

- 10) **Handshakes:** Students enter the room one at a time. As each student enters it is a new stage. Each student shakes hands with every other student in the room once (no repeats). How many handshakes?

1 student, 0 handshakes 2 st, 1 hs 3 st, 2 hs _____

Sequence Rule: _____

| stage (x) | total shakes (y) |
|-----------|------------------|
|-----------|------------------|

Number of students in your class _____

_____ (total handshakes)

equation: $y =$ _____

equation family _____