

LessonTitle: Fractal Functions		Alg 5.8
Utah State Core Standard and Indicators Algebra Standards 2, 4 Process Standards 1-5		
Summary		
In this lesson, students build fractals and track the growth of fractal measurements using tables and equations.		
Enduring Understanding <ul style="list-style-type: none"> • Most mathematics you study in school is old knowledge. But mathematicians and scientists currently research the geometry of fractals to understand nature’s forms and patterns! • Fractal development is “Endlessly repeating, Ever diminishing, Self similar 		Essential Questions What are the patterns of fractal development and how can we describe them using Algebra?
Skill Focus <ul style="list-style-type: none"> • Function patterns and equations • Scaling • Self-similarity • Geometric transformation, complexity, and limits. 		Vocabulary Focus
Assessment		
Materials: Isometric and regular dot paper, Calculators, Computers, video tapes ordered from the district		
Launch		
Explore		
Summarize		
Apply		

Directions: Students and teachers know little about fractals. As in all lessons the discussion or hook activity is important. We suggest:

- Activities which give students experience with similarity. Try “Growing Generations of Similar Figures.” Access from Algebra module 2, Activity 2.4a. Or see below.

- Have students build larger and larger self similar figures using linking cubes.
- Show a video clip from Jurassic Park in which they talk about fractals and chaos.
- To introduce the idea of “Endlessly repeating, Ever Diminishing, Self Similar,” demonstrate an animated fractal building process. (Can be found in Geometer’s Sketchpad version 4, Sketches folder/ Geometry/ Fractal Gallery.)
- Show a video tape. There are 3 presently owned by JSD on fractals.

Have students research the web to answer specific questions about fractals. See the Fractal Research cooperative power point activity below.

Students create fractals on isometric and plain graph paper. Then track the development (growth or shrinking) toward a limit.



Extensions: 1) Students play the chaos game. 2) Students create fractals using Geometer’s sketchpad or a drawing program. 3) Students research fractals on the internet and create power point presentations. See all below.

Growing Generations of Similar Figures




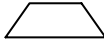

Name _____

Build growing generations of similar figures. (the same shape but larger)

Use the same pattern block units for each succeeding generation. For example, how many square pattern blocks will you use to create the next larger square for each generation? How many triangle blocks will you use to create the next larger triangle, etc.

Example: Generation  Generation 2 

Hint: You may need to think in terms of hexagon areas, not hexagons.

Generation	 Units of area	 Units of area	 Units of area	 Units of area	 Units of area
1	1	1	1	1	1
2					
3					
4					
5					
20					

- 1) How can you know if the generation you are building is similar (by mathematical definition) to the preceding generations?

- 2) What is the rule you would use to figure out the 20th generation without building the generation?

- 3) “What happens to area when you double dimensions of a given polygon? Triple them?”

- 4) Explain all you learned about similar figures. Be certain to talk about angles and ratios of corresponding sides.

Alg 5.8 Fractal Function I. The Sierpinski Gasket

Draw an equilateral triangle with a side length of 32 units on isometric dot paper. This is Stage 0. To build a 3rd stage Sierpinski Triangle, repeat the following 3 times.

Fractal Building Algorithm for the Sierpinski Triangle

- Locate the midpoints of each side of the triangle.
- Connect the midpoints to create four similar triangles within the larger triangle.
- Throw away the center.
- Repeat the process in the remaining triangles

Complete the table below.

Stage	No. of Triangles	No. of Holes	New Triangle Area	Total Area	New Triangle Perimeter	Total Perimeter
0	1	0	1	1	3	3
1	3	1				

Number of Triangles

Predict the number of triangles in a 10th stage Sierpinski Triangle. _____ What did you do to find that prediction?

Create a formula for the nth stage. Number of triangles (t) = _____

At what stage in the Sierpinski triangle will the number of sub-triangles first be large enough to assign a separate one to each and every person on earth? SHOW your thinking.

Area of Triangles

Predict the remaining area in a 6th stage Sierpinski Triangle. _____ What did you do to find that prediction?

Create a formula for the nth stage. Triangle area (a) = _____

As the stage gets larger without bound, what happens to the shaded area?

Questions:

1) If the edge of a stage 0 triangle is 1 foot, will the perimeter ever get large enough to exceed the circumference of the earth? Why or why not?

2) What would remain of the original large triangle after four iterations if the algorithm were changed to keeping only the inner triangle? Explain.

Extra Credit: Develop formulas for Holes, Total Area, Total Perimeter.

Fractal Function II. The Square Carpet

Draw a square with a side length of 27 units on regular dot paper. This is Stage 0. To build a 4th stage square carpet, repeat the following algorithm 3 times.

Fractal Building Algorithm for Square Carpet

- Trisect each side of the square.
- Connect the trisection points to create 9 squares within the square.
- Throw away the center.
- Repeat the process in the remaining triangles

Complete the table below.

Stage	No. of Squares	No. of Holes	New Square Area	Total Area	New square Perimeter
0	1	0	1	1	4
1	8	1			

Develop formulas for:

No. of Squares (s) = _____ No. of Holes (h) = _____

New Square Area (A_n) = _____ Total Area (A_t) = _____

New Square Perimeter (P_n) = _____

Questions:

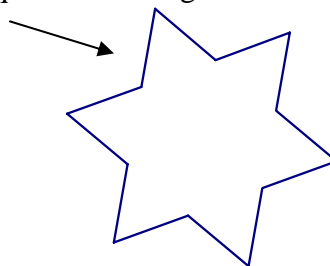
1) If this iterated (repeated) process were to continue without end, what would happen?

2) Suppose the algorithm were changed from keeping the eight border sub-squares to keeping only the four corner sub-squares. What figure would emerge after two iterations?

Fractal Function III. The Koch Snowflake

On a piece of isometric dot paper center a drawing of an equilateral triangle that is 27 units on each side. That is stage 0. Stage 1 looks like this.

- Describe the building algorithm



To create a Koch Snowflake, repeat the algorithm 4 times and complete the table below.

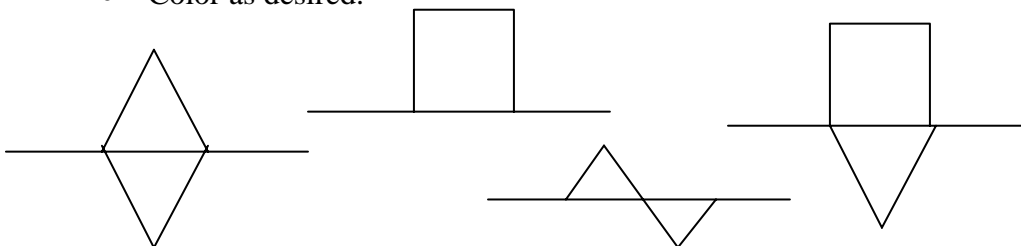
Stage	Total Area (fraction)	Total Area (decimal)	Perimeter (fraction)	Perimeter (decimal)
0	1		3	
1				
2				
3				
4				
10				
n				

- When will the snowflake become too large for the paper?
- What happens to the area?
- What happens to the perimeter?
- Describe your ideas about what a “limit” might mean in mathematics.
- Were there any limits in the Sierpinski Gasket or the Square Carpet? If so describe them.

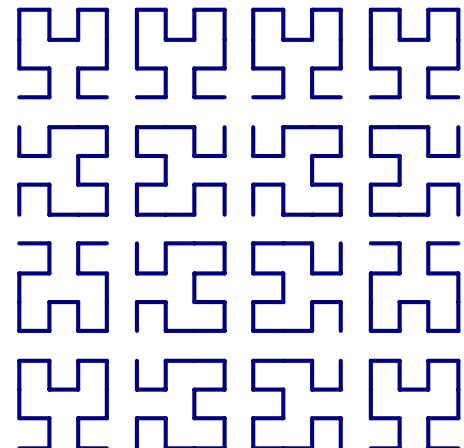
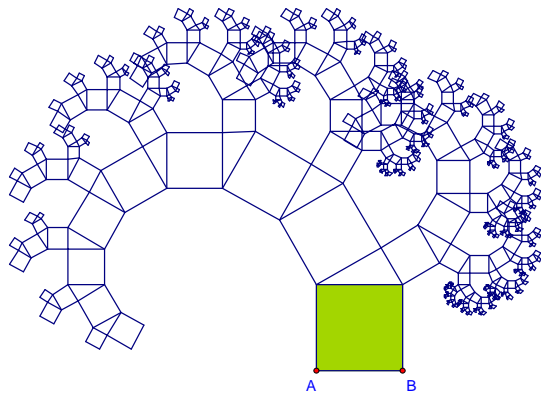
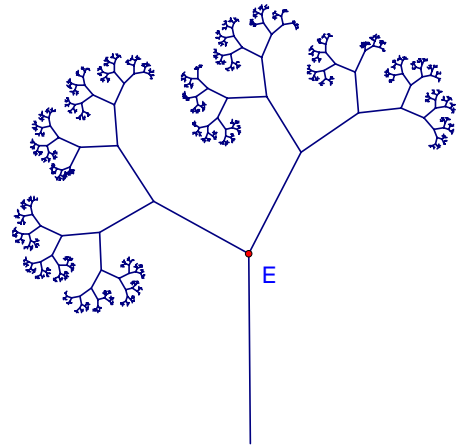
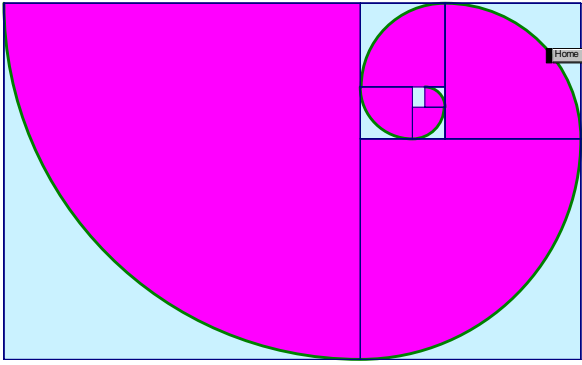
IV. Your OWN Koch-like Curve.

Draw a line on isometric graph paper, the long way—making certain the line can be divided into 3 or 4 equal segments. This is stage 0. Then....

- Divide the line into segments.
- Create your curve pattern.
- Iterate for each new line segment.
- Repeat the iteration until you are satisfied with the end product.
- Color as desired.



V. Are They Fractals? Why or why not?



The Chaos Game

Play this game and watch the apparent chaotic behavior of a moving point. Begin doing this by hand. Then continue using the program on the calculator.

Start with any point inside the triangle formed by vertices L, T, and R.

Step 1 Roll the die and move according to these rules.

For 1 or 2, move halfway to L.

For 3 or 4, move halfway to T.

For 5 or 6, move halfway to R.

Step 2 Connect the point to this newly located midpoint.

Step 3 Starting from the last midpoint located, repeat the steps.

Continue repeating the process extending the path to four successive midpoints.

● T

L ●

● R

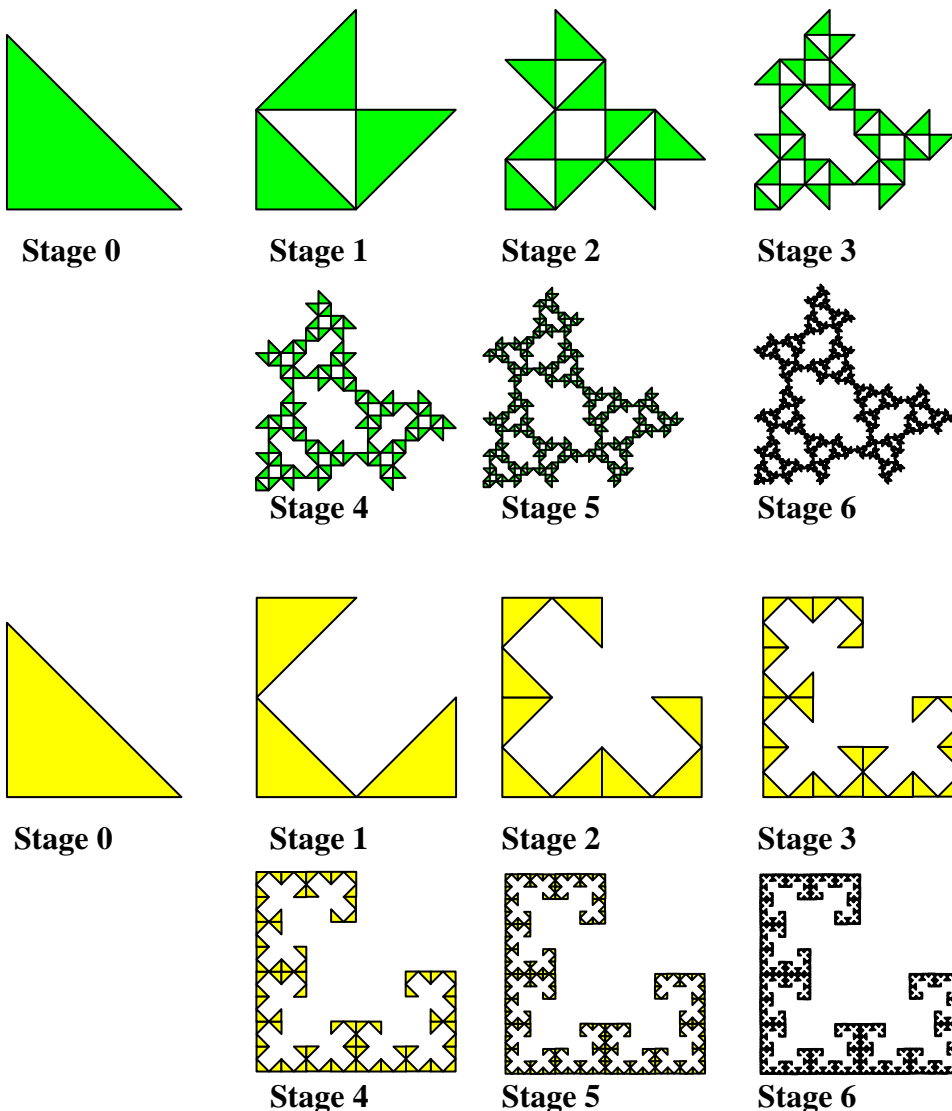
Creating a Fractal on Geometer's Sketchpad.

Go to the help menu and follow the steps. Directions vary according to the version of Geometer's Sketchpad you're using.

Creating a Fractal using a drawing program.

In Microsoft office, go to draw and turn on a grid. Then...

- Create the identity, stage 0. (see examples below)
- The building algorithm:
 - Step 1: Reduce the dimensions by half.
 - Step 2: Replicate and translate. Copy, paste and translate the identity twice. (slide, rotate, reflect--see examples below)
 - Step 3: Rebuild. Select the three triangles, go to draw and group them. Start again at Step 1.
- Repeat as many stages as desired



Fractal Research

A cooperative Power Point Presentation

After your group has received your research question,

- Open a new Power Point Presentation.
- Write your question and the names of those in your group.
- Access the internet to research information about fractals.
- Create your presentation slides to answer your question.
- Create one slide to show your favorite fractal from those you observed on the fractal sites.
- Your research presentation slides will be evaluated for quality not quantity.

1. What are fractals?
2. Why study fractals?
3. What are the mathematics of fractals?
4. Where are fractals in the world?
5. What is the history of fractals
6. What is fractal research used for?
7. What are scientists and mathematicians learning from fractals?
8. How are chaos and fractals related?
9. How do you make a fractal?