Graphing Quadratic Functions

Summary
In this lesson, students explore the transformations of the graph $y = x^2$ using graphing calculator and simply plotting points. Identifying the vertex and line of symmetry is emphasized throughout the lesson. Students have an opportunity to graph a quadratic function, given the equation, and also to write the equation of a quadratic function, given the graph of the function.

Utah State Core Standards

- Find the vertex, maximum or minimum values, intercepts, and axis of symmetry of a quadratic or absolute value function, algebraically, graphically, and numerically.
- Sketch the graph of a quadratic and absolute value function.
- Write an equation of a parabola in the form $y = a(x - h)^2 + k$ when given a graph.
- Perform the transformations of stretching, shifting, and reflecting the graphs of linear, absolute value, quadratic, and radical functions.

Desired Results

Benchmark/Enduring Understanding

Students will understand the graphs of quadratic functions and use their knowledge of transformations to predict and quickly sketch the graphs.

Essential Questions	Skills
What does the graph of $y = x^2$ look like? What are the common features of the graphs of quadratic functions? How can we quickly sketch the graph of a quadratic function?	Graphing by plotting points. Graphing by transformation—vertical stretch, vertical shift, reflection, horizontal shift. Writing the equation of the graph of a quadratic function.

Assessment Evidence

There are understanding checks in the lesson at the end of each section. The final question prompts students to generalize their understanding from the lesson and create an example that illustrates their explanation.

Instructional Activities

Launch: Teacher and class work together to generate and confirm the graph of $y = x^2$.

Explore: Students may work individually or in pairs or groups to complete the worksheet. The teacher should monitor student progress by checking for understanding at the end of each section.

Summarize: Student responses to the final question should be discussed in class

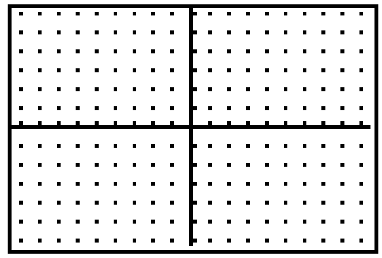
so that their understanding of quadratic transformations is confirmed. Students should practice and extend their knowledge of quadratic transformations using quick-draws (available on this site) as a follow-up to this lesson.
Materials Needed
Copies of worksheet Graphing calculator

Graphing Quadratic Functions

I. The parent function $y = x^2$

Use a table to plot points and graph: $y = x^2$. Show your work here:

x	$y = x^2$	(x, y)
0		
1/2		
-1/2		
1		
-1		
2		
-2		
3		
-3		



Use your graphing calculator to confirm your graph. This shape is called a parabola.

What is the domain of this function? What is the range?

Domain: ___________________ Range: ________________________

Write at least three features you notice about the graph of a parabola and **explain how** that feature is related to the equation.

1.

2.

3.

The minimum point of this parabola is called the vertex. What are the coordinates of the vertex of this parabola?

The imaginary line that goes through the vertex and cuts the parabola in half is called "the line of symmetry." What is the <u>equation</u> of this line?

II. Transformations

From your work with previous functions like $y = \sqrt{x}$ and $y = |x|$, you have learned that you can use the graph of the parent function to graph other equations of the same form.

A. Vertical Shifts

If I wanted to shift the graph of the parent function, $y = x^2$, up 2 units without changing its shape, what would be the equation of the parabola?

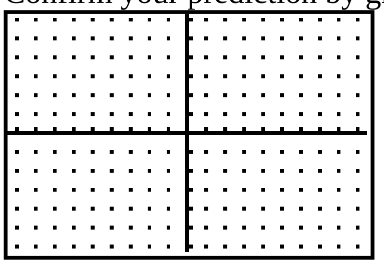
Use your calculator to confirm or adjust your equation.

What are the coordinates of the vertex?

What is the domain and range of this function?

What do you predict the graph of $y = x^2 - 3$ will look like?

Confirm your prediction by graphing on your calculator and draw the graph (accurately) here.



What is the equation of the line of symmetry for this parabola?

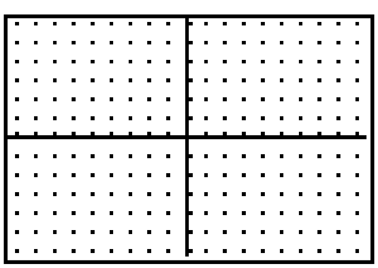
B. Reflections

Predict the equation of a parabola that is the same size, same shape, and has the same vertex as $y = x^2$, but is opening downward (reflected over the *x*-axis)

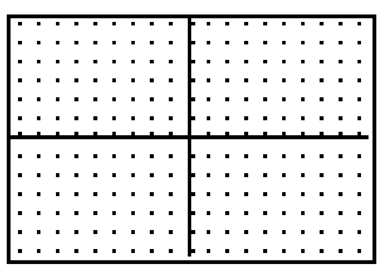
Use your calculator to confirm or adjust your prediction.

Graph these parabolas accurately without your calculator, then use your calculator to check and adjust your graphs, if necessary.

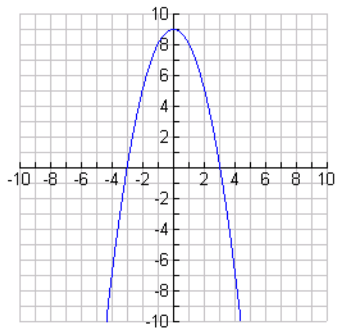
$y = -x^2 + 6$



$y = x^2 - 4$



Write the equation of the parabola that is shown here:



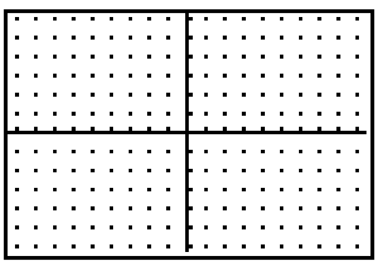
Equation: ________________________

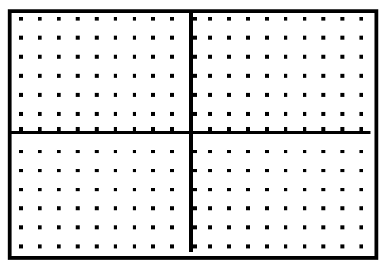
C. Vertical Stretch

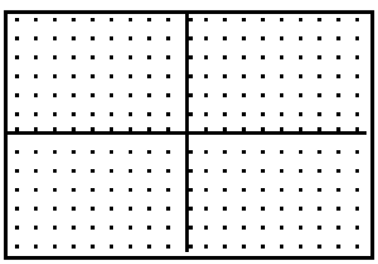
Use your calculator to graph the following equations in the same window:

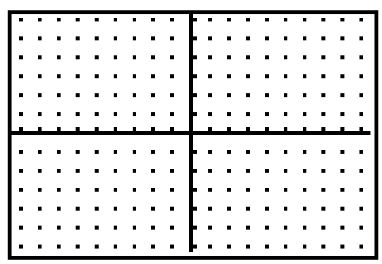
a) $y = x^2$ b) $y = 1.5x^2$ c) $y = 2x^2$ d) $y = 3x^2$

Show your graphs here. Draw the graphs accurately, being especially careful to get the correct values at x = 1 and x = -1. Use the tables of values on your calculator to help you.

a) 

b) 

c) 

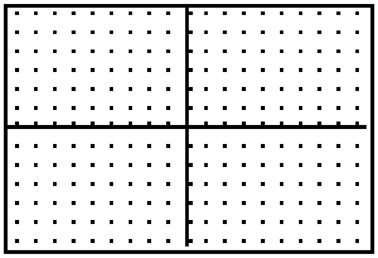
d) 

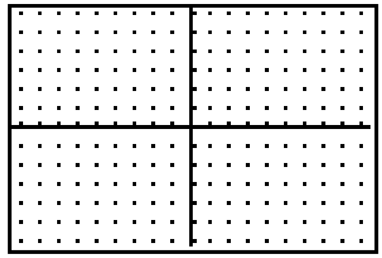
Compare each function with $y = x^2$. What is the difference between $y = x^2$ and $y = ax^2$ when $a > 1$? Explain why this is the case.

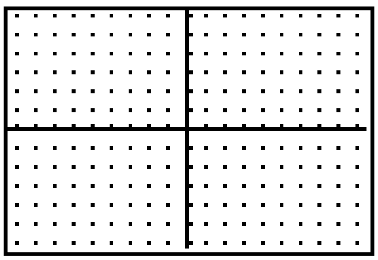
Use your calculator to graph the following equation in the same window:

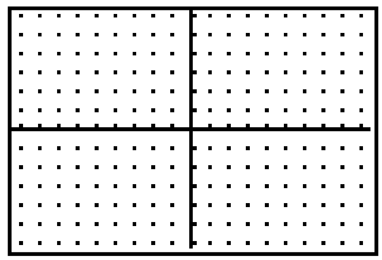
a) $y = x^2$ b) $y = \frac{3}{4}x^2$ c) $y = \frac{1}{2}x^2$ d) $y = \frac{3}{2}x^2$

Show your graphs here. Draw the graphs accurately, being especially careful to get the right values at x = 1 and x = -1.

a) 

b) 

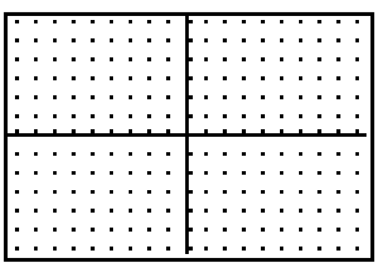
c) 

d) 

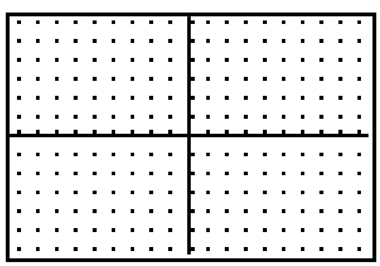
What is the difference between $y = x^2$ and $y = ax^2$ when $0 < a < 1$? Explain why this is the case.

Graph the following equations, first without your calculator. Then use your calculator to check your graphs. Make corrections if necessary.

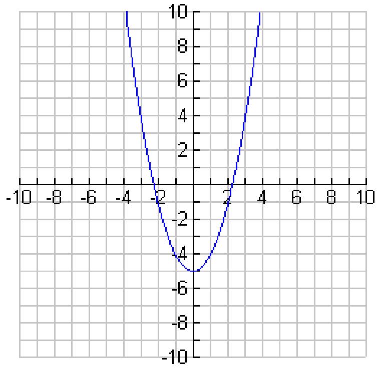
$y = 2x^2 - 3$



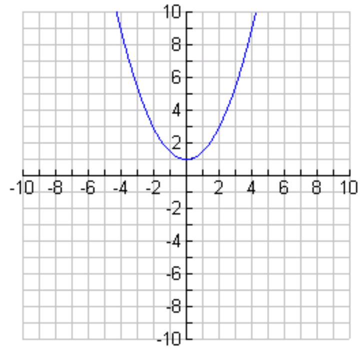
$y = -\frac{1}{2}x^2 + 3$



Write the equations of the graphs shown here. Use your calculator to check your equation. Be very careful to be sure that that the graph from your equation is identical to the one shown here.



Equation: ______________________



Equation: ________________________

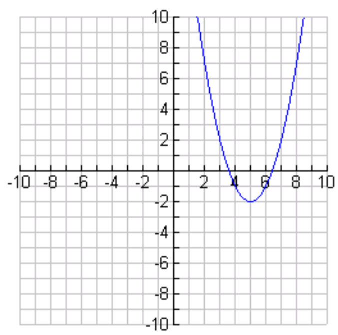
E. Horizontal Shifts

The final transformation that we must be able to handle is the horizontal shift. Predict the equation of the parabola that comes from shifting the parent function $y = x^2$ to the right 2 units.

Check your prediction using your calculator.

Now predict and check the equation of the parabola that comes from shifting the parent function, $y = x^2$, to the left 2 units.

Write the equation of the parabola shown. Identify the vertex and the equation of the line of symmetry.



Equation: _____________________

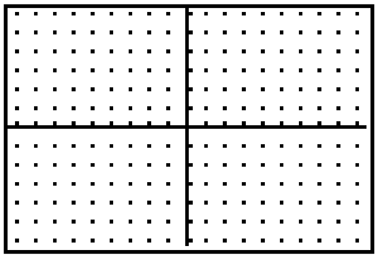
Vertex: __________

Line of Symmetry: __________

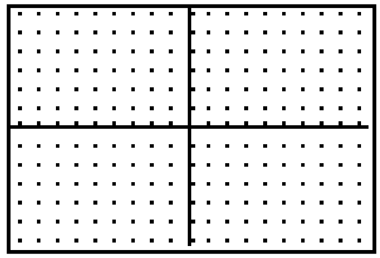
III. Putting it all together.

Identify the transformation to the parent function, $y = x^2$ in the following equations. Graph each function without a calculator, then, check your graphs with your calculator.

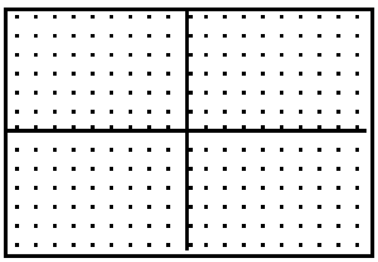
$y = (x+2)^2 - 3$



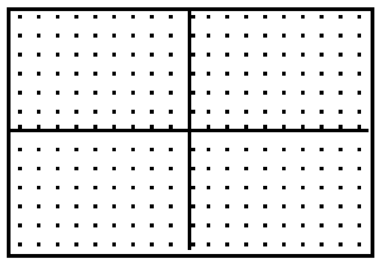
$y = -(x-1)^2 + 4$



$y = 2(x-3)^2 - 1$



$y = -\frac{1}{2}(x+2)^2$



When we write a general rule in math, we replace the numbers with letters, so that the letters can stand for any number. The general equation of a parabola, in vertex form is:

$y = a(x-h)^2 + k$

Use this equation to explain how to identify the transformations of the parent function, $y = x^2$. Be sure to include how you can tell if the parabola opens up or down and where the vertex and the line of symmetry will be located. Create at least one example to illustrate your understanding.