Absolute Value Equations and Inequalities

Summary
This lesson systematically develops the idea of absolute value as distance from zero on the number line, and uses understanding of this concept to solve single-variable absolute value equations and inequalities.
Utah State Core Standard
Standard 2, Objective 2.2 Evaluate, solve, and analyze mathematical situations using algebraic properties and symbols. • Solve first-degree absolute value equations. • Solve single-variable quadratic and absolute value inequalities. • Represent intervals with correct symbolic notation; e.g., $a < x < b$, (a, b), [a, b]. Standard 3, Objective 3.2 Specify locations and describe spatial relationships using coordinate geometry. • Sketch the solutions of absolute value and quadratic inequalities of one variable on a number line.
Desired Results
Benchmark/Enduring Understanding
Students will understand that absolute value is the distance from zero on a number line.

Essential Questions	**Skills**
What does absolute value mean?	Solving single-variable absolute value equations and inequalities. Representing inequalities on a number line and algebraically.

Assessment Evidence
After completing this lesson, students will demonstrate their ability to represent absolute value expressions algebraically, verbally, or on a number line. Students will solve absolute value equations and inequalities and represent the solutions on a number line and with correct algebraic notation.

Instructional Activities
Launch: Ask students what they know about absolute value. Extend their understanding by explaining that absolute value makes the number positive because it is the distance from zero on the number line. Explore: Students work through the lesson on the worksheet individually or in groups (recommended). It is useful to stop and be sure that all students are on the right track after each section. Summarize: Ask students to write a procedure for solving absolute value

inequalities.
Materials Needed
Copies of Worksheet

Absolute Value Equations and Inequalities

I. The absolute value of a number is defined to be its distance from zero on the number line. For example, $|3| = 3$ and $|-3| = 3$ because both 3 and -3 are 3 units away from zero on the number line.

A. With this definition in mind, then the equation $|x| = 3$ is asking us to find all the numbers that are 3 units away from zero on the number line. To solve the equation, we have to know that absolute value means distance, and then to find values for x that make the equation true.

What are the solutions to the equation $|x| = 3$?

Graph the solutions on a number line.

B. In words, what is the equation $|x| = 5$ asking us to find? What are the solutions to the equation?

Graph the solutions on the number line.

C. In words, what is the equation $|x| = -6$ asking us to find?

Is this possible? What would you say about the solution to this equation?

II. Now, we can get a little tricky by adding a number inside the absolute value.

A. What does this equation mean, in words?
$|x - 2| = 3$.

We can't solve this equation if it still contains the absolute value symbols. Use the meaning of absolute value to write and solve <u>two</u> equations that replace this equation.

Solve the two equations and check your solutions to see if they work in the original equation.

Graph your solutions on the number line.

B. Solve and graph your solutions to the following equations:

a) $|x-5|=3$ *b*) $|x+1|=3$ *c*) $|x+4|=3$

How do these equations relate to the equation: $|x|=3$?

If I describe the solution to $|x|=3$ in words, I would say, "All the numbers 3 units from zero on the number line." Write a similar statement that describes the solution to equations *a*, *b*, and *c* above.

a)

b)

c)

III. The same ideas can be used to solve inequalities with absolute value. To describe the solutions to the inequality $|x|>3$, what would you say (in words)?

Graph your solutions on a number line.

Use correct notation to name the intervals.

A. Find the solutions and graph the inequality: $|x|>6$

B. Explain why absolute value inequalities of the type $|x|>b$ will generally have two intervals in the solution. How can you find the two intervals for this type of inequality?

IV. What if we add numbers inside the absolute value?

A. Write the meaning, in words, of the inequality: $|x+4|>5$

Use the meaning to write two inequalities that are equivalent to the original statement.

Explain how these two inequalities relate to the original absolute value statement.

Solve the two inequalities:

Graph the solutions on one number line.

Do the solutions make sense in the original absolute value inequality? Explain.

B. Use the meaning of the inequality $|x-6|\geq 7$ to break it into two inequalities, solve and graph on a number line.

Check your solutions before going on.

C. Use this technique to solve and graph the following inequalities:

a) $|x+4|>2$ b) $|2x+1|\geq 5$ c) $|3x-2|>4$

V. Absolute value inequalities that are "less than" work similarly.

A. If I were given the inequality $|x|<3$, I could say, "All the numbers that are less than 3 units away from 0 on the number line." Draw a number line and graph the solution to this inequality.

Write the solution to the inequality by naming the interval of numbers that make the inequality true. Use correct notation.

B. Based on your work with absolute value equations, how do you think the inequality $|x-2|<3$ is related to $|x|<3$?

Find the solution and graph the inequality by separating the absolute value inequality into two statements and solving as you did with the previous examples. :

How wide is the solution interval?

What number is the center of the solution interval?

Write a distance statement that describes the solution set for this inequality.

C. Find the solution and graph the inequality: $|x-5|<2$

How wide is the solution interval?

What number is the center of the solution interval?

How is the inequality $|x-5|<2$ related to $|x|<2$?

Write a distance statement that describes the solution set for this inequality.

D. Solve and graph each absolute value inequality, using the method of your choice.

a) $|x-3| \leq 6$ b) $|x+4| \leq 1$ c) $|2x-3| < 5$

Write the absolute value inequality that corresponds to the following description:

a) All the numbers that are less than 8 units away from 1 on the number line.

b) All the numbers that are at least 6 units away from 2 on the number line.

c) All the numbers that are no more than 13 units away from -7 on the number line.

d) All the numbers that are greater than 20 units away from -5 on the number line.