Grade 5 ~ Benchmark 1 ~ Instructional Support

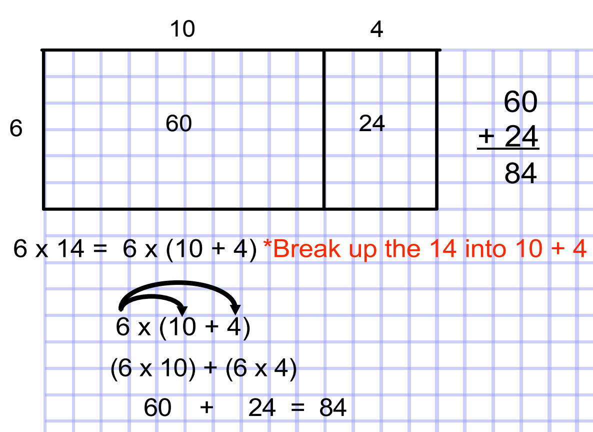
Block 1	Block 2	Block 3
Data Students solidify knowledge of data collection processes and creating various data representations. **Emphasis is put on *analyzing* the data: *proposing and justifying inferences.***	**Decimals: Representations, Compare/Order, + -** Students expand place value from one million (fourth grade) to one billion for whole numbers and from hundredths (fourth grade) to thousandths for decimals. Representations should include models and symbols. (Decimal models are illustrated on **4th Grade Benchmark 2 Instructional Support** page.)	**Multiplication** Students solidify multiplication with whole numbers (distributive property, traditional algorithm). Students also learn to use the various symbols for multiplication. * *Instructional Support materials use the various symbols.*

Place Value

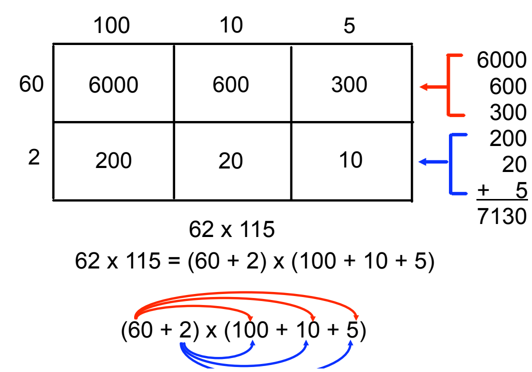
Use the students' knowledge of fractions to launch the topic of decimal numbers. Decimals are a way to write tenths (e.g., 8/10 = 0.8). Similarly, hundredths can be written 23/100 = 0.23, and thousandths can be written 658/1,000. **Marking the whole number place value is significant**: it helps the students remember that there are no 'whole' units; decimals are a fractional unit. More specifically, decimals are an extension of the base-ten number system to express an amount that is less than one. Decimals are 'base-ten' fractions. It is a good idea to teach the students that the fraction bar is also a division sign. Each decimal value is 10 times less (smaller) than its value to the left. Thus, entering 8 ÷ 10 in a calculator, the answer .8 appears (generally without marking the whole number place value). Entering 0.8 ÷ 100 will yield the answer .008. In block 3, students begin to multiply decimals. This lays the foundation for multiplying fractions in a very controlled manner – students **ONLY** work with base-ten fractions. The Utah Math Core further limits this introductory work to **tenths**.

The Distributive Property of Multiplication

It is very important for students to become more fluent in using the distributive property to solve multiplication problems. The distributive property is foundational understanding for Algebraic reasoning and computing; it is often modeled using arrays. This is the information about the distributive property that is found on the **4th Grade Benchmark 2 Instructional Support** page.



Arrays can be created for 3-digit by 2-digit multiplication problems using base-ten blocks (or on graph paper). Students should understand that
~the blocks maintain the value of the number in expanded form, and
~after the number is expanded, all parts of one factor should be multiplied by all parts of the other factor.



Open arrays need not be proportional once the concept is understood, and they can be created to multiply numbers of any size.

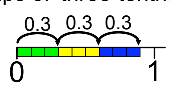
<u>Teaching Sequence for Multiplying Decimals:</u> While students develop ideas of decimal multiplication, it is important to allow them time to practice **reading, interpreting,** and then **making** the measurement/number line and area/regional models ('set' models are not 'user-friendly' for decimals). This is a skill that will transfer to many other mathematical ideas such as multiplying and dividing fractions. Also, students should be developing 'symbol vocabulary' ~ reading and using the various symbols for multiplication.

Stage 1: whole number • tenths (e.g., 3 • 0.3)

This is natural and intuitive because it follows the familiar interpretation of multiplication (skip-counting on a number line; 'groups of'; repeated addition). **A measurement model** is shown, but students can also use tenth strips to represent these problems.

3 • 0.3 = 0.9

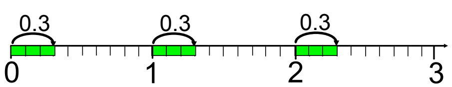
three 'groups of' three-tenths is nine-tenths



Students may also begin to utilize the commutative property; however, the model will look different and require the students to shift their understanding of how to read the multiplication sign (see Stage 2).

0.3 • 3 = 0.9

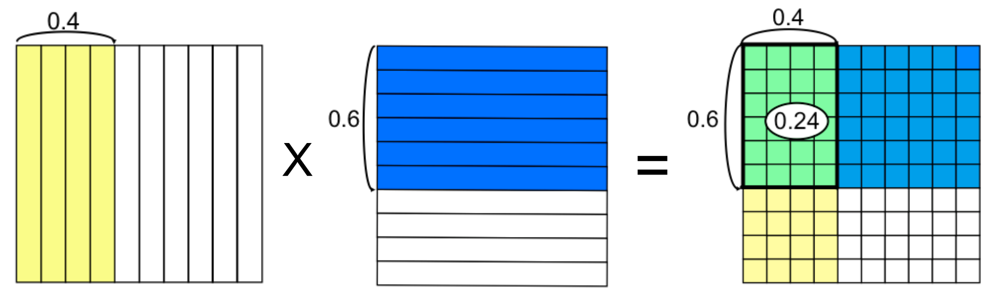
three-tenths 'of' three is nine-tenths



*** When students are comfortable with problems that result in a product that is less than 1, encourage them to move beyond the '1' mark (e.g., How would 3 ● 0.7 be shown?).

Stage 2: tenth • tenths (e.g., 0.4 • 0.6)

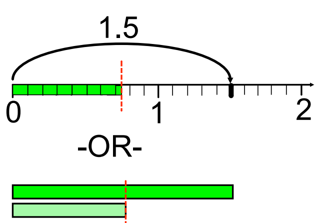
This stage requires a conceptual shift in the understanding of multiplication. Now, rather than finding 'groups of' a given size, students are finding *portions* of a given size ~ AND that 'given size' is also a *portion*: they are finding ***a portion of a portion***. This can be shown on a number line or by using an **area/regional model** which is shown below (tenth strips and hundredth grids).



*** In this stage, it is easy to make a connection between decimals and benchmark fractions. Though multiplying fractions is developed in Benchmark 3, it is appropriate to include fractions in classroom discussions at this point (*if you feel your class is ready for it*) or small group discussions (*for students who are ready for it*). Begin by using a very familiar benchmark fraction for them such as ½ = 0.5.

Stage 3: whole number w/ tenths • tenths (e.g., 0.5 • 1.5)

<u>Measurement Model</u> It is most helpful to use decimals that are also benchmark fractions (halves and fourths) when working with a measurement model.

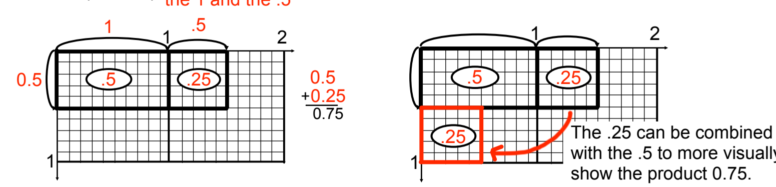


<u>Area Model</u> The **area model** most clearly shows the distributive and commutative properties of multiplication, and it makes sense of the algorithm. Students can clearly see the parts .25 and .5 that need to be combined. This problem was specifically chosen because it uses decimals that are also benchmark fractions (halves and fourths).

0.5 • 1.5 = 0.75

0.5•(1 + .5) break apart the 1.5

0.5•(1 + .5) distribute the 0.5 to the 1 and the .5



Grade 5 ~ Benchmark 2 ~ Instructional Support

Block 4	Block 5	Block 6
2-D Geometry & Measurement Emphasis is on analyzing attributes and properties of two-dimensional shapes and measurement of two-dimensional shapes. Perimeter measures **distance *around*** a shape. Area measures the amount needed to ***cover*** a shape. The 4th Grade Benchmark 3 Instructional Support page contains support for reviewing and extending these concepts.	**Division & Algebra** Emphasis is on interpreting remainders, numeric patterns of dividing decimals by 10, 100, and 1,000, and learning the various symbols for division.	**Number Theory, Integers, Coordinate Graphing** Emphasis is on • developing strategies, including rules of divisibility, to determine if a number is prime, composite, or neither. • representing integers (number lines, coordinate graphs) • probability

Division & Algebra

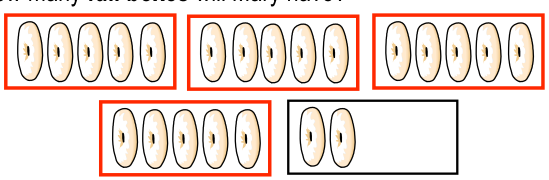
Students should become familiar with using the various symbols for division. These include the fraction bar, ÷, and ⟌ .

Division Interpretations

Consider the different answers to 22 ÷ 5. Answers include 4 R 2, 4 2/5, 4.4, 4 or 5, 4, and 5. In fourth grade, students learned the Remainder Quotient Theorem (see **4th Grade Benchmark 1 Instructional Support** page). In this Theorem, students do not need to consider "What do I do with that remainder? What does it mean?". In 5th grade, they will examine the different interpretations of remainders. What contexts determine how the remainder is interpreted? The answers 4, 5, and 4 or 5 are used contexts where the object being divided generally ***can not*** be 'cut apart' into smaller pieces (e.g., people, furniture, cars). The answers 4 2/5 and 2.2 are generally used in contexts where the object being divided ***can*** be cut apart (e.g., pizza, brownies, measurements). Examples are provided below. Interpretations heavily depend on context. It is helpful to vary the problem structures presented to students. Problem structures and examples are found on the next page.

22 ÷ 5 = 4

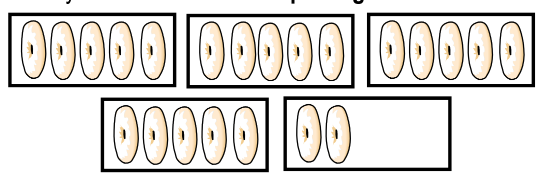
Context: Mary has 22 donuts. Five donuts will fill one box. How many **full boxes** will Mary have?



4 boxes will be full.

22 ÷ 5 = 5

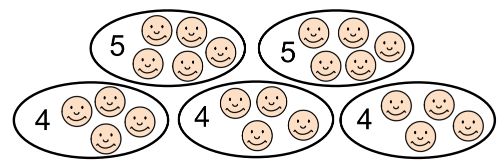
Context: Mary has 22 donuts. Five donuts will fill one box. How many boxes are needed **to package all** of the donuts?



5 boxes are needed.

22 ÷ 5 = 4 or 5

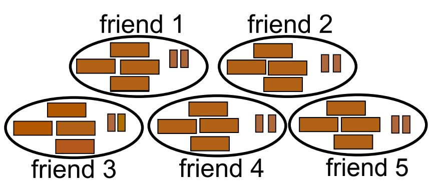
Context: There are 22 students going on a field trip. There are 5 chaperones. How many students can be in a group?



4 or 5 students can be in a group.

22 ÷ 5 = 4 2/5

Context: Five friends are equally sharing 22 brownies. How many brownies will each friend get?



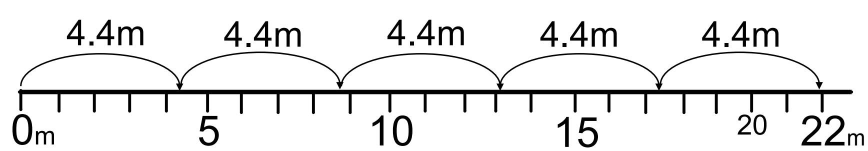
Each friend can get 4 whole brownies; the remaining 2 brownies can be split into fifths; each friend will get 2/5.

Each friend will get 4 2/5 brownies.

** This quotient will also work with the example given for 22 ÷ 5 = 4; Mary will have 4 2/5 boxes.

22 ÷ 5 = 4.4

Context: Marcia has 22 meters of ribbon to make 5 bows for the carnival. How much ribbon will Marcia use for each bow?



Each bow will use 4.4 meters of ribbon.

** Metric units are good to use because it is a base-ten measurement system.

Grade 5 ~ Benchmark 3 ~ Instructional Support

Block 7	Block 8	Block 9
Fractions: Representations Emphasis is on the *conceptual* understanding of fractions. Represent fractions on a number line, in a set, and part of an area. The measurement and region models are very useful for modeling fractions greater than one and mixed numbers.	**Geometry & Measurement** Emphasis is on finding the volume and surface area of three- dimensional shapes. This is an extension of geometry and measurement concepts from block 4.	**Fractions: Operations** Emphasis is on adding, subtracting and multiplying fractions. For adding and subtracting, students need to be comfortable finding a common denominator. For multiplication, the focus is on *conceptual* understanding, not just a procedure.

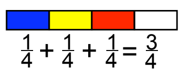
3 Models of Fractions (**See Grade 3 Benchmark 3 Instructional Support** page)

Suggested sequence for Teaching Multiplication of Fractions Measurement and regional models best illustrate the meaning of multiplying fractions. (Parker & Baldridge, 2008, Elementary Math for Teachers, p. 144-145)

1 – Whole Number x Fraction

This is natural and intuitive because it follows the familiar interpretation of multiplication ("groups of", repeated addition, rectangular arrays).

$3 \bullet \frac{1}{4} = 3 \text{ 'groups of' } \frac{1}{4} = \frac{3}{4}$



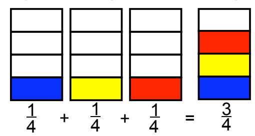
The Commutative Property holds true: the product is three-fourths. The illustration will look different (see stage 2).

2 – Fraction x Whole Number

Similar to multiplying decimals, this step requires a conceptual shift in the understanding of multiplication.

It is interpreted as taking a fractional part ***of*** a unit (drop the word 'groups').

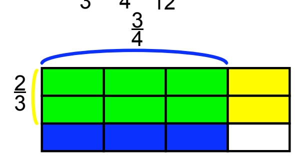
$\frac{1}{4} \bullet 3 = \frac{1}{4} \text{ 'of' } 3 = \frac{3}{4}$



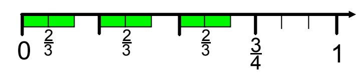
3 – Fraction x Fraction

This step continues to interpret multiplication as "of" (not 'groups of') and uses a regional model that is subdivided in both directions.

$\frac{2}{3} \bullet \frac{3}{4} = \frac{6}{12}$



$\frac{2}{3} \bullet \frac{3}{4} = \frac{6}{12}$ *Can also be illustrated on a measurement model.

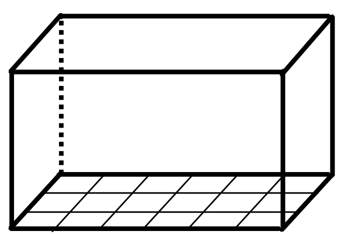


Volume

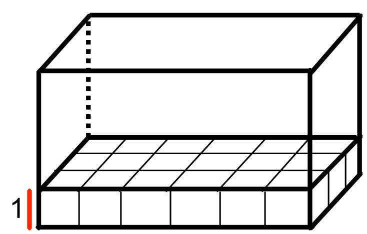
Volume is 'filling' a 3-dimensional object. How much will 'it' hold without gaps or overlaps? Cubic units are used to measure volume. It is very important that students experience the concrete-pictorial-abstract cycle of learning as they begin studying volume. Engage students in tasks that allow them to build 3-dimensional objects using the idea of layering. Allowing them time to develop the skill of drawing a 3-dimensional cube and rectangular prisms. **Last**, they should be able to develop a formula based on their concrete and pictorial work. This is an example of an activity that helps students develop a concrete visual of volume.

Task: Volume of a Box

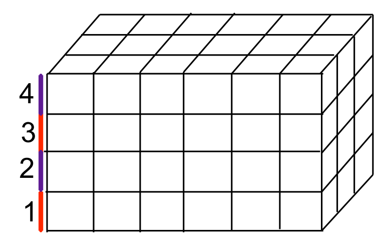
One way to help children make sense of volume is through the use of cardboard boxes (shoe, cereal, etc.) The task for students is to fill the box with some cubes (enough to make a row or two). Then they are asked to decide how many cubes the box will hold. They may use rulers and calculators to help with computations. It is possible that the cubes used for this activity will not fill the box precisely. In that case, students can ignore the fractional portions, since the purpose of this activity is to help students connect to the idea that finding volume is asking how many of a certain unit it takes to fill the box.



Base is 3 x 6. Area of base is 18 squares.



Base holds a set of 18 cubes.



Four layers of 18 cubes fills the box. V = 4 x 18 cubes