

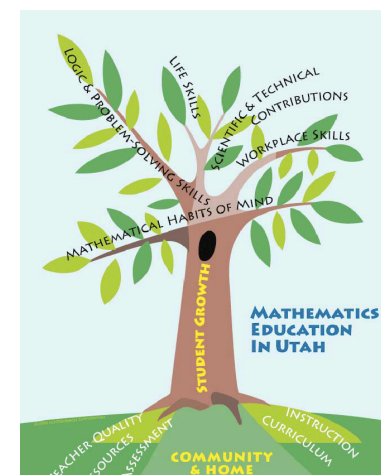


# JORDAN SCHOOL DISTRICT Curriculum & Staff Development

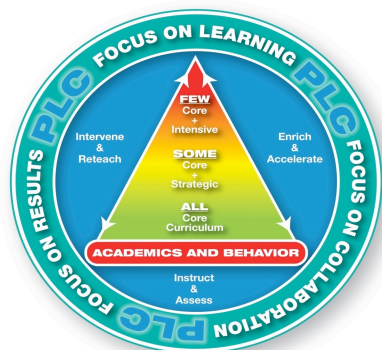
## Elementary Mathematics

The Jordan School District Mathematics Team supports the Utah State Office of Education Mathematics mission:

*The mission of mathematics education in Utah is to promote student growth and learning in mathematics in order to prepare students to thrive and contribute in the global economy of the 21st century.*



JORDAN SCHOOL DISTRICT **J** MULTI-TIERED PLC SUPPORT SYSTEM



The Mathematics Team also supports the Jordan School District Curriculum Department mission:

*The Curriculum and Staff Development Department exists to provide support and professional development for administrators, teachers, and the school community in evidence-based best practices through a collaborative, interdisciplinary, and unified approach for increased student achievement and organizational effectiveness.*

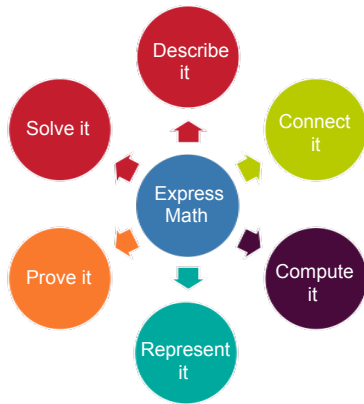
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# Jordan School District Scope and Sequence

## Math Expressions



Allowing students differentiated ways of accessing and expressing mathematics increases engagement and successful acquisition of concepts and skills. Students **describe** mathematics, **connect it** to other subjects within and outside of mathematics, **compute** answers with and without technology, **represent** mathematics in multiple ways, **solve** problems, and **prove** mathematical conjectures. (USOE, 2010, *Utah's 3-Tier Model of Mathematics Instruction*)

## Scope and Sequence Format

- Overview: This page identifies the mathematical content to be studied in each block. It serves as a year-at-a-glance up to the CRT's.
- Benchmark
  - Instructional Support
 

This page contains models, illustrations, and explanations specific to the content of a Benchmark. It is intended to be a resource for explaining key mathematical ideas. The instructional methods presented provide tools students can use to demonstrate and communicate their mathematical ideas and understanding.
  - Blocks
 

In most cases, the core content of a block page has been selected based on connections that can be made between content standards. Though blocks can be taught in any order, the sequence presented provides a coherent flow to mathematics instruction throughout the year. There are three columns and one footer-row on a block page.

    - ⇒ Core Content is stated in the first column. This is the Utah State Elementary Mathematics Core Curriculum that must be taught. Words/phrases that appear with ~~strike-out~~ indicate that that topic is not a focus for that block.
    - ⇒ Teaching Strategies are listed in the middle column. It consists of a few teaching strategies aligned to core content. It is recommended that teachers use their own effective teaching strategies in addition to those provided.
    - ⇒ Resources are listed in the last column. It is **NOT** a checklist for instruction. Teach the **Core**, not the resources. It is also NOT an all-inclusive list. (Not all Utah Core content is covered in the district-adopted resources.) It is recommended that teachers use good resources.
    - ⇒ The footer-row identifies Core and additional vocabulary. Core vocabulary *must* be taught; additional vocabulary can help scaffold student understanding. This row also identifies the specific assessment that is available for that block.

## Assessments

- Pre-Assessment: These tests are specific to grade-level content. They can be used as an annual pre- and post-test. They are intended to provide immediate feedback regarding students' aptitude and existing knowledge of the mathematics of that grade.
- Block: These are 1) specific to the block content, and 2) intended to be used as formative assessments that provide immediate feedback that can be used for tiered-interventions for individual students. Item-analysis can also identify concepts for which the whole-class needs a re-teaching lesson. Block assessments can be administered in a variety of ways (e.g., whole-class at end of block; a few problems per day; small-group). Block assessments are also intended to provide opportunity to explicitly teach test-taking strategies.
- Benchmark: These are 1) cumulative for *that specific* Benchmark, 2) summative in that scores are reported to the district and content from that Benchmark will not be repeated on subsequent Benchmarks, and 3) formative in that continued interventions should take place for students not mastering content.
- CRT Review: These are cumulative items that can be used to review content and/or test-taking strategies before the CRT. Best practices recommend that the items be reviewed in smaller amounts (e.g., 5-7 problems per day) over several weeks.

## Third Grade Scope and Sequence Overview

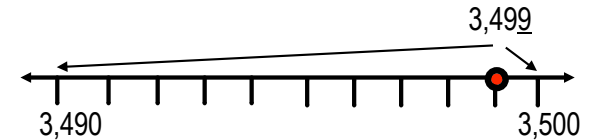
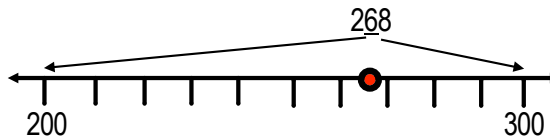
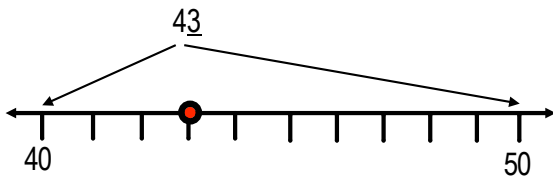
Block	Focus
1	<p style="text-align: center;"><b>Base Ten &amp; Data</b> <b>Representing and Estimating Numbers and Data</b></p> <p><i>During this block, students learn to collect, organize, and display data in a variety of ways (tables, charts, graphs). Students extend their knowledge of base ten numeration to include whole numbers up to 10,000. They continue identifying specific place value of a given number and then estimate by rounding to the nearest ten, hundred, and thousand. Teachers also use this time to pre-assess students and to help them maintain fluency with basic addition and subtraction facts.</i></p>
2	<p style="text-align: center;"><b>Operations, Equivalence &amp; Measurement</b> <b>Adding, Equivalence, and Time</b></p> <p><i>During this block, students solidify understanding addition and its properties. They extend computation to include larger numbers and money. Student work includes measuring time.</i></p>
3	<p style="text-align: center;"><b>Operation Meaning/Relationships &amp; Measurement</b> <b>Subtraction, Weight, and Length</b></p> <p><i>During this block, students examine part-whole relationships as applied to subtraction and measurement. They use ideas of estimation and equivalence to extend subtraction skills to include larger numbers. Students engage in identifying measurable attributes, specifically weight and length. They quantify these attributes using customary and metric units of measure.</i></p>
4	<p style="text-align: center;"><b>Operation Meanings/Relationships &amp; Measurement</b> <b>Representing Multiplication and Capacity</b></p> <p><i>During this block, students study the meaning, models, and properties of multiplication. They begin learning facts with 0, 1, 2, 5,9, and 10 as factors. Students continue identifying measurable attributes of objects adding units of capacity.</i></p>
5	<p style="text-align: center;"><b>Operation Meanings/Relationships &amp; Geometry</b> <b>Representing Multiplication and 1- and 2- Dimensional Geometry</b></p> <p><i>During this block, students study the meaning, models, and properties of multiplication. They begin learning facts with 3, 4, and 6 as factors. Students learn 1- and 2-Dimensional attributes and look at part-whole relationships to describe, compare and/or classify shapes.</i></p>
6	<p style="text-align: center;"><b>Operation Meanings/Relationships &amp; Measurement</b> <b>Representing Multiplication and Figuring Perimeter</b></p> <p><i>During this block, students study the meaning, models, and properties of multiplication. They begin learning facts with 7 and 8 as factors. Students study geometry focusing on perimeter.</i></p>
7	<p style="text-align: center;"><b>Operation Meanings/Relationships &amp; Geometry</b> <b>Representing Division and Geometric Equivalency (Congruency)</b></p> <p><i>During this block, students study the meaning and models of division. Particular emphasis should be placed on the inverse relationship between multiplication and division and the two models of division (partitive, or 'sharing', and quotitive, or 'repeated subtraction'). Students study geometry in motion to determine congruency.</i></p>
8	<p style="text-align: center;"><b>Operation Meanings/Relationships &amp; Chance</b> <b>Representing Division and Probability</b></p> <p><i>During this block, students study the meaning and models of division. Particular emphasis should be placed on the inverse relationship between multiplication and division and the two models of division (partitive, or 'sharing', and quotitive, or 'repeated subtraction'). Students experiment with outcomes of probability.</i></p>
9	<p style="text-align: center;"><b>Equivalence &amp; Comparison of Fractions</b></p> <p><i>During this block, students study the meaning and models of fractions using halves, thirds, fourths, sixths, and eighths. Students develop understanding for equivalency using concrete models and pictorial representations.</i></p>

## Grade 3 ~ Benchmark 1 ~ Instructional Support

Block 1	Block 2	Block 3
<p><b>Number Sense &amp; Data</b></p> <p>1) Collect and organize data in a variety of ways.                      2) Base 10 numeration including whole numbers to 10,000 (<b>place value and rounding</b>).                      3) Maintain fluency with basic addition and subtraction facts.</p>	<p><b>Addition Properties and Time</b></p> <p>1) Solidify understanding of <b>addition</b> and its <b>properties</b>.                      2) Extend computation to larger numbers and money.                      3) Student work includes measuring <b>elapsed time</b>.</p>	<p><b>Subtraction and Measurement</b></p> <p>1) Compute and solve problems involving 3 and 4 digit numbers, using a variety of strategies.                      2) Identify and measure <b>length and weight</b> using customary and metric units.</p>

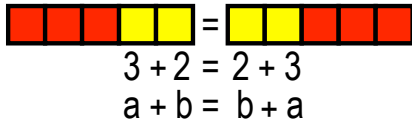
### Number Sense

Rounding: To round a number simply means to substitute a 'nice' number that is close so that computation can be done more easily. A number line is most useful in helping students select 'nice' numbers that are close to the target number.

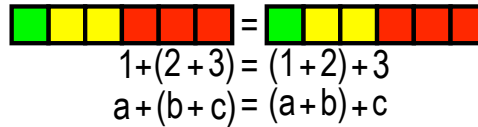


### Addition Properties:

#### Commutative Property



#### Associative Property



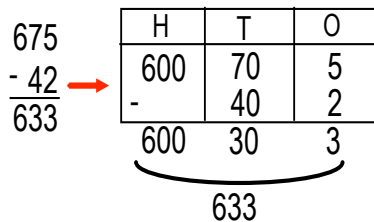
#### Identity Property of Addition

$$5 + 0 = 5$$

$$n + 0 = n$$

### Subtraction:

#### Expanded Form



#### Compensation

$$\begin{array}{r}
 400 \\
 - 26 \\
 \hline
 \end{array}
 \begin{array}{l}
 \text{take one away} \\
 \text{from both}
 \end{array}
 \begin{array}{r}
 399 \\
 - 25 \\
 \hline
 374
 \end{array}$$

\*\*\* More than '1' can be taken away from both numbers. This strategy emphasizes the *distance* between two points on a number line. It can also be used for addition.

#### Traditional Algorithm

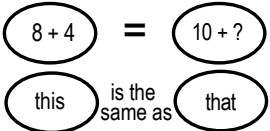
$$\begin{array}{r}
 3 \ 17 \\
 247 \\
 - 28 \\
 \hline
 219
 \end{array}$$

\* Note: During measurement students will be asked to measure to a basic fraction of halves, thirds, and fourths (a second grade core review). You may want to review halves and fourths on a number line with students. A number line model of fourths is found on the 3<sup>rd</sup> Benchmark 3 page.

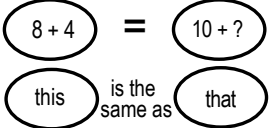
**Grade 3 ~ Block 1 ~ Number Sense & Data**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>            Objective 1: Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.            a. Read, write, and represent whole numbers using standard and expanded form.            c. Identify the place and the value of a given digit in a four-digit numeral and round numbers to the nearest ten, hundred, and thousand.            d. Order and compare whole numbers on a number line and use the symbols <math>&lt;</math>, <math>&gt;</math>, <math>\neq</math>, and <math>=</math> when comparing whole numbers.</p> <hr/> <p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>            Objective 1: Create, represent, and analyze growing patterns.            a. Create and extend growing patterns using objects, numbers, and tables.            b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.</p> <hr/> <p><b>Standard 5: Students will collect and organize data to make predictions and identify basic concepts of probability.</b>            Objective 1: Collect, organize, and display data to make predictions.            a. Collect, read, represent, and interpret data using tables, graphs, and charts, including keys (e.g., pictographs, bar graphs, frequency tables, line plots).            b. Make predictions based on a data display.</p>	<p>Establish classroom routines.</p> <p>Provide time for students to practice basic addition and subtraction facts</p> <p>The number line is a model used extensively throughout the core. Help students become proficient at</p> <ul style="list-style-type: none"> <li>• <b>reading</b> a number line,</li> <li>• <b>interpreting</b> a number line, &amp;</li> <li>• <b>making</b> a number line.</li> </ul> <hr/> <p>Identifying and extending patterns is an essential algebra skill. Begin the year by using growing patters that include <b>addition</b>. <i>Later in the year</i>, students will find growing patterns that include <b>multiples</b>.</p> <hr/> <p>Use data concepts to learn about each other (e.g., collect data about favorite book, how students get to school, # of siblings, etc...).</p>	<p><b>2008 SFAW Diamond Edition</b>            1-2 Numbers in the Hundreds            1-3 Place Value Patterns            1-4 Numbers in the Thousands            1-5 Greater Numbers            1-7 Comparing Numbers            1-8 Ordering Numbers            1-9 Number Patterns            1-10 Rounding Numbers</p> <hr/> <p>4-5 Using Tally Charts to Organize Data            4-6 Using Line Plots to Organize Data            4-7 Reading Pictographs and Bar Graphs            4-8 Writing to Compare</p>
<p><b>Core Vocabulary:</b> data, table, chart, graph, frequency table, line plot, pictograph, bar graph, growing pattern, greater than, less than, equal to, <math>&lt;</math>, <math>&gt;</math>, =,  <b>Additional Vocabulary:</b> expanded form, standard form, estimate, round, period, number sentence, symbol</p>		<p><b>Assessment:</b>  <b>Block 1</b></p>

**Grade 3 ~ Block 2 ~ Addition (computation & properties) and Measurement (time)**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 1:</u> Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.                      b. Demonstrate multiple ways to represent numbers using models and symbolic representations (e.g., fifty is the same as two groups of 25, the number of pennies in five dimes, or <math>75 - 25</math>).  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.                      a. Use a variety of methods to facilitate computation (e.g., estimation, mental math strategies, paper and pencil).                      b. Find the sum or difference of numbers, including monetary amounts, using models and strategies such as expanded form, compensation, partial sums, and the standard algorithm.</p>	<p>Continue with key ideas from previous blocks as needed by students.</p>	<p><b>2008 SFAW Diamond Edition</b>                      2-2 Relating Addition and Subtraction                      2-4 Problem Solving Strategy: Write a Number Sentence                      2-7 Estimating Sums                      2-8 Overestimates &amp; Underestimates                      3-3 Adding Three-Digit Numbers                      3-4 Adding Three or More Numbers</p>
<p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.                      b. Solve equations involving equivalent expressions (e.g., <math>6 + 4 = \Delta + 7</math>).                      c. Use the <math>&gt;</math>, <math>&lt;</math>, and <math>=</math> symbols to compare two expressions involving addition and subtraction (e.g., <math>4 + 6 \square 3 + 2</math>; <math>3 + 5 \square 16 - 9</math>).                      d. Recognize and use the commutative, associative, and identity properties of addition.</p>	<p>The equal sign can also be read as "the same as". This is helpful when determining equivalent expressions.                      Example:  </p>	<p>2-1 Addition Properties                      2-3 Find a Rule</p>
<p><b>Standard 4: Students will select and use appropriate units and measurement tools to solve problems.</b>  <u>Objective 1:</u> Select and use appropriate tools and units to estimate and measure length, weight, capacity, time, and perimeter of two-dimensional figures.                      d. Identify the number of minutes in an hour, the number of hours in a day, the number of days in a year, and the number of weeks in a year.  <u>Objective 2:</u> Solve problems involving measurements.                      a. Determine simple equivalences of measurements (e.g., 90 min. = 1 hr. 30 min.).                      d. Determine elapsed time in hours (e.g., 7:00 a.m. to 2:00 p.m.).</p>		<p>4-3 Elapsed Time                      4-4 Using a Calendar</p>
<p><b>Core Vocabulary:</b> sum, expressions, equations, expressions, inequalities  <b>Additional Vocabulary:</b> elapsed</p>		<p><b>Assessment:</b>                      Block 2</p>

**Grade 3 ~ Block 3 ~ Subtraction (computation) and Measurement (length & weight)**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4 digit numbers and basic facts of multiplication and division.                      a. Use a variety of methods to facilitate computation (e.g., estimation, mental math strategies, paper and pencil).                      b. Find the sum or difference of numbers, including monetary amounts, using models and strategies such as expanded form, compensation, partial sums, and the standard algorithm.</p>	<p>Continue with key ideas from previous blocks as needed by students.</p> <p>Use a balance of subtraction models.</p> <p>Provide time for students to practice basic addition and subtraction facts</p>	<p><b>2008 SFAW Diamond Edition</b>                      3-6 Regrouping                      3-7 Subtracting Two-Digit Numbers                      3-8 Models for Subtracting Three Digit Numbers                      3-9 Subtracting Three-Digit Numbers                      3-10 Subtracting Across Zeros                      3-12 Adding and Subtracting Money                      3-13 Choose a Computation Method</p>
<p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.                      b. Solve equations involving equivalent expressions (e.g., <math>6 + 4 = \Delta + 7</math>).                      c. Use the <math>&gt;</math>, <math>&lt;</math>, and <math>=</math> symbols to compare two expressions involving addition and subtraction (e.g., <math>4 + 6 \square 3 + 2</math>; <math>3 + 5 \square 16 - 9</math>)</p>	<p>The equal sign can also be read as "the same as". This is helpful when determining equivalent expressions.                      Example:  </p>	<p>3-14 Equality and Inequalities                      3-15 Problem Solving Applications                      12-3 Problem Solving Strategy: Working Backwards</p>
<p><b>Standard 4: Students will select and use appropriate units and measurement tools to solve problems.</b>  <u>Objective 1:</u> Select and use appropriate tools and units to estimate and measure length, weight, capacity, time, and perimeter of two-dimensional figures.                      a. Describe the part-whole relationships (e.g., 3 feet in a yard, a foot is <math>\frac{1}{3}</math> of a yard) between metric units of length (i.e., centimeter, meter), and among customary units of length (i.e., inch, foot, yard), <del>capacity</del> (i.e., cup, quart), and weight (i.e., pound, ounce).                      b. Measure the length of objects to the nearest centimeter, meter, half- and quarter-inch, foot and yard.                      c. <del>Measure capacity using cups and quarts, and</del> measure weight using pounds and ounces.  <u>Objective 2:</u> Solve problems involving measurements.                      a. Determine simple equivalences of measurements (e.g., 30 inches = 2 feet and 6 inches; <del>6 cups = 1 1/2 quarts; 90 min. = 1 hr. 30 min.</del>).                      b. Compare given objects according to measurable attributes (i.e., length, weight, <del>capacity</del>).</p>		<p>9-12 Length                      9-13 Measuring to the Nearest <math>\frac{1}{2}</math> and <math>\frac{1}{4}</math> Inch                      9-14 Length in Feet and Inches                      9-15 Feet, Yards, and Miles                      9-16 Problem Solving Skills: Extra or Missing Information                      10-6 Centimeters and Decimeters                      10-7 Meters and Kilometers                      12-4 Customary Units of Weight</p>
<p><b>Core Vocabulary:</b> difference, expanded form, greater than, less than, equal to, <math>&lt;</math>, <math>&gt;</math>, <math>=</math>, measure, unit, metric system, customary system, pound, ounce, weight, length, centimeter, meter, inch, foot, yard  <b>Additional Vocabulary:</b> estimating, rounding, period, standard form, attribute</p>	<p><b>Assessment:</b>                      Block 3                      Benchmark 1 (reported)</p>	

## Grade 3 ~ Benchmark 2 ~ Instructional Support

This **ENTIRE** benchmark is dedicated to students developing ideas of multiplication: meaning, models and illustrations. They use properties and reasoning strategies to help build understanding and fluency with basic multiplication facts 0-10 (please see the 'Math Fact Mastery Help' resource on the website). As they study multiples of a specific factor, they can connect the patterns to algebraic growing patterns via input-output tables and other strategies.

Block 4	Block 5	Block 6
<p><b>Meaning and Models of Multiplication &amp; Capacity</b></p> <p>1) Students begin with using 'skip counting' patterns learned in 1<sup>st</sup> and 2<sup>nd</sup> grades: 2's, 5's, and 10's. Facts with zero and one can be learned using the Identity and Zero Properties of multiplication.</p> <p>2) Students use tools to measure capacity.</p>	<p><b>Meaning and Models of Multiplication &amp; Geometry</b></p> <p>1) Students extend facts to include factors of 3, 4, and 9.</p> <p>2) Students extend ideas of shape classification by using side lengths and right angles to classify polygons.</p>	<p><b>Meaning and Models of Multiplication &amp; Perimeter</b></p> <p>1) Students extend facts including factors of 6, 7, and 8. Research shows that 6x7, 6x8, 7x8, 7x9, and 8x9 are the most troublesome facts.</p> <p>2) Students begin conceptual understanding of perimeter.</p>

### **Multiplication of Whole Numbers**

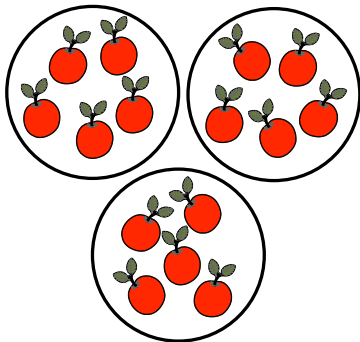
repeated addition of **same-sized groups**

### **Multiplication Models**

It is important for understanding that students see all 3 models early and often, and learn to use them when solving word problems; it is the variety of models and contexts that builds understanding – **NOT** the difficulty of the problem. (Parker & Baldrige, 2008, p. 25-26)

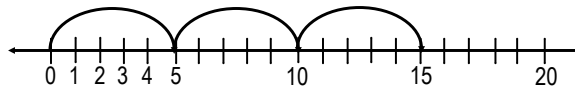
#### **Set Model**

3 x 5 is interpreted as '3 groups of 5 objects'



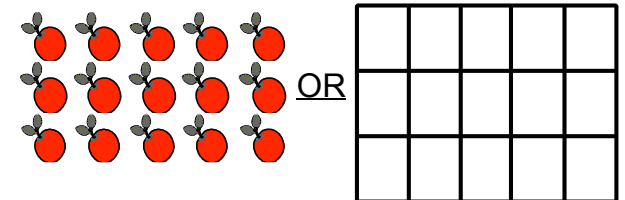
#### **Measurement Model**

3 x 5 is illustrated as '3 hops on the number line, each of 5 lengths'



#### **Array Model**

3 x 5 is illustrated as '3 rows of 5 objects'



\*\*The illustration on the right is frequently called the 'area model'. Students begin to make a connection between multiplication and measuring area using *arrays*; *area* is not introduced until 4<sup>th</sup> grade.

### **Multiplication Properties**

#### **Multiplicative Identity**

$$5 \times 1 = 5$$

$$n \times 1 = n$$

#### **Commutative Property**

$$6 \times 7 = 7 \times 6$$

$$a \times b = b \times a$$

\*\* Best shown using an array model.

#### **Associative Property**

$$3 \times (1 \times 4) = (3 \times 1) \times 4$$

$$a \times (b \times c) = (a \times b) \times c$$

#### **Zero Property**

$$5 \times 0 = 0$$

$$n \times 0 = 0$$

#### **Distributive Property**

Students often use the distributive property when using known facts to help them find unknown facts. For example, students may use a 'double and one more set' strategy for facts with a 3: e.g., 3 x 7 I know double 7 is 14; one more set of 7 is 21. Algebraically, this is  $3(2 + 1)$  OR  $3(2) + 3(1)$ . This strategy can be used with any fact.

**Grade 3 ~ Block 4 ~ Multiplication (models & meaning) and Measurement (capacity)**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 1:</u> Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.                      e. Identify factors and multiples of whole numbers.  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).                      b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.                      d. Demonstrate the effect of place value when multiplying whole numbers by 10.                      e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.                      c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).</p>	<p>Use all of the multiplication models to help students represent the meaning of multiplication.</p>	<p><b>2008 SFAW Diamond Edition</b>                      5-1 Multiplication as Repeated Addition                      5-2 Arrays and Multiplication                      5-3 Writing Multiplication Stories                      5-5 2 as a Factor                      5-6 5 as a Factor                      5-7 10 as a Factor                      5-9 Multiplying with 0 and 1                      5-11 Practicing Multiplication Facts</p>
<p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 1:</u> Create, represent, and analyze growing patterns.                      b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.                      d. Recognize and use the commutative, associative, distributive, or identity properties of addition and multiplication, and the zero property of multiplication.</p>	<p>Input/Output tables can be used to help students find patterns that are based on multiples.</p>	
<p><b>Standard 4: Students will select and use appropriate units and measurement tools to solve problems.</b>  <u>Objective 1:</u> Select and use appropriate tools and units to estimate and measure length, weight, capacity, time, and perimeter of two-dimensional figures.                      a. Describe the part-whole relationships (e.g., 3 feet in a yard, a foot is 1/3 of a yard) between metric units of length (i.e., centimeter, meter), and among customary units of length (i.e., inch, foot, yard), capacity (i.e., cup, quart), and weight (i.e., pound, ounce).                      c. Measure capacity using cups and quarts, and measure weight using pounds and ounces.  <u>Objective 2:</u> Solve problems involving measurements.                      a. Determine simple equivalences of measurements (e.g., 6 cups = 1 1/2 quarts)                      b. Compare given objects according to measurable attributes (i.e., length, weight, capacity).</p>		<p>12-1 Customary Units of Capacity</p>
<p><b>Core Vocabulary:</b> factor, product, array, multiple, measure, unit, metric system, customary system, capacity, commutative, associative, distributive, and identity properties of addition and multiplication, zero property of multiplication,  <b>Additional Vocabulary:</b> cups, pints, quarts, gallons</p>		<p><b>Assessment:</b>                      Block 4</p>

**Grade 3 ~ Block 5 ~ Multiplication and Geometry**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 1:</u> Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.                      e. Identify factors and multiples of whole numbers.  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).                      b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.                      e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.                      c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).</p> <p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 1:</u> Create, represent, and analyze growing patterns.                      b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.                      d. Recognize and use the commutative, associative, distributive, and identity properties of addition and multiplication, and the zero property of multiplication.</p> <p><b>Standard 3: Students will describe and analyze attributes of two-dimensional shapes.</b>  <u>Objective 1:</u> Describe and compare attributes of two-dimensional shapes.                      a. Identify, describe, and classify polygons (e.g., pentagons, hexagons, octagons).                      b. Identify attributes for classifying triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).                      c. Identify attributes for classifying quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).                      d. Identify right angles in geometric figures, or in appropriate objects, and determine whether other angles are greater or less than a right angle.</p>	<p>Continue with key ideas from previous blocks as needed by students.</p> <p>Use all of the multiplication models to help students represent the meaning of multiplication.</p> <hr/> <p>Input/Output tables can be used to help students find patterns that are based on multiples.</p> <hr/>	<p><b>2008 SFAW Diamond Edition</b>                      5-10 9 as a Factor                      6-1 3 as a Factor                      6-2 4 as a Factor</p> <hr/> <p>8-4 Lines and Line Segments                      8-5 Angles                      8-6 Polygons                      8-7 Triangles                      8-8 Quadrilaterals</p>
<p><b>Core Vocabulary:</b> factor, product, array, multiple, polygon, attribute, quadrilateral, equilateral triangle, isosceles triangle, right triangle, pentagon, hexagon, octagon, parallel, right angle  <b>Additional Vocabulary:</b></p>	<p><b>Assessment:</b>                      Block 5</p>	

**Grade 3 ~ Block 6 ~ Multiplication and Perimeter**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 1:</u> Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.                      e. Identify factors and multiples of whole numbers.  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).                      b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.                      e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.                      c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).  <b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 1:</u> Create, represent, and analyze growing patterns.                      b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.                      d. Recognize and use the commutative, associative, distributive, and identity properties of addition and multiplication, and the zero property of multiplication.  <b>Standard 4: Students will select and use appropriate units and measurement tools to solve problems.</b>  <u>Objective 1:</u> Select and use appropriate tools and units to estimate and measure length, weight, capacity, time, and perimeter of two-dimensional figures.                      e. Describe perimeter as a measurable attribute of two-dimensional figures, and estimate and measure perimeter with metric and customary units.  <u>Objective 2:</u> Solve problems involving measurements.                      c. Solve problems involving perimeter.</p>	<p>Continue with key ideas from previous blocks as needed by students.</p> <p>Use all of the multiplication models to help students represent the meaning of multiplication.</p> <hr/> <p>Input/Output tables can be used to help students find patterns that are based on multiples.</p> <hr/>	<p><b>2008 SFAW Diamond Edition</b>                      6-3 6 and 7 as Factors                      6-4 8 as a Factor                      6-5 Practicing Multiplication Facts                      6-7 Using Multiplication to Compare                      6-8 Patterns on a Table                      6-10 Find a Rule</p> <hr/> <p>8-11 Perimeter</p>
<p><b>Core Vocabulary:</b> factor, product, array, multiple, perimeter  <b>Additional Vocabulary:</b> dimension</p>		<p><b>Assessment:</b>                      Block 6                      Benchmark 2 (reported)</p>

## Grade 3 ~ Benchmark 3 ~ Instructional Support

Block 7	Block 8	Block 9
<b>Division &amp; Geometry Transformations</b> 1) Students study the meaning of division through contexts and models. Emphasis is made on the inverse relationship between multiplication and division (the 'doing' and 'undoing'). 2) Students learn how to 'move', or transform geometric figures to determine congruency.	<b>Division &amp; Probability</b> 1) Students continue the study of division and its relationship to multiplication. 2) Students conduct experiments to qualitatively ( <b>use words instead of numbers</b> ) describe the probability of outcomes. They learn to use the words <i>certain</i> , <i>impossible</i> , <i>likely</i> , <i>unlikely</i> to describe probability.	<b>Fractions</b> Students study the meaning and models of benchmark fractions: halves, thirds, fourths, sixths, and eighths. Equivalency ideas are developed using concrete models and pictures. They compare and order fractions.

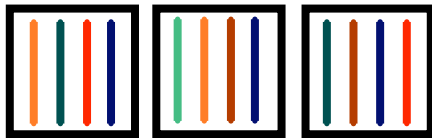
### Division

Division is the inverse operation of multiplication ~ it 'un-does' the putting together of equal-sized groups. Division begins with the total items and then requires students to find the *number of groups (measurement division)* or the *size of the groups (partitive division)*. Students have considerable life experience with finding the *size of the group (partitive division)*. Consider: Three friends want to share six cookies. How many cookies will each friend get? Six represents the total cookies, and three friends represent the *number of groups*; students must find the *size of the group* or 'how many cookies will each friend get'? Students best develop understanding of the **meaning** of division through a variety of rich, contextual problems. (Parker & Baldrige, 2008, p. 32)

#### 2 Interpretations of Division

##### 1- Partitive Model

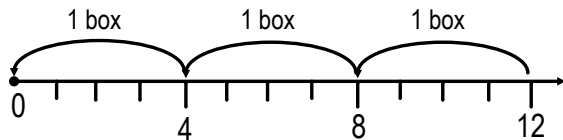
Sam has 12 markers to put away. There are 3 boxes. How many markers will go in each box? (Like dealing cards, students 'deal out' the markers to each box; 12 items in 3 groups.)



4 markers will go in each box.

##### 2- Measurement Model

Sam has 12 markers to put away. Four markers will fit in a box. How many boxes will he need? (Like skip counting, students 'count by' 4's until they get to 12. They could also use repeated subtraction by 4's.)

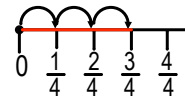


$$\begin{array}{r}
 12 \\
 - 4 \leftarrow 1 \text{ box} \\
 \hline
 8 \\
 - 4 \leftarrow 1 \text{ box} \\
 \hline
 4 \\
 - 4 \leftarrow 1 \text{ box} \\
 \hline
 0 \quad 3 \text{ boxes}
 \end{array}$$

#### Fractions

Fractions represent a *relationship between a part and its whole*. Fractions can be represented by using a measurement model (which includes the number line and a bar diagram), a set model, and a regional/array model.

##### Measurement Model



Bar Diagram  
(thickened number line)

##### Set Model

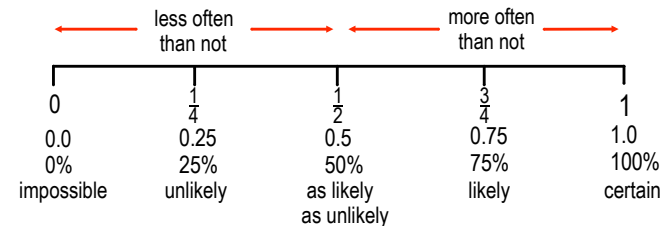


##### Regional/Array Model



#### Probability

The probability of an event can be a number from 0 to 1. It can be expressed as a fraction, a decimal or a percent. If the probability of an event is 0, it is *impossible*. If an event is *certain*, it has a probability of 1. The more *unlikely* an event, the closer it is to 0; the more *likely* an event, the closer it is to 1 (Cavanagh, 2000, p. 292).



**Grade 3 ~ Block 7 ~ Division and Geometry: Transformations**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers</b>  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).                      b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.                      c. Demonstrate, using objects, that multiplication and division by the same number are inverse operations (e.g., (e.g., <math>3 \times \square = 12</math> is the same as <math>12 \div 3 = \square</math> and <math>\square = 4</math>).  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.                      c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).</p> <hr/> <p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.</p> <hr/> <p><b>Standard 3: Students will describe and analyze attributes of two-dimensional shapes.</b>  <u>Objective 2:</u> Demonstrate the meaning of congruence through applying transformations.                      a. Demonstrate the effect of reflection, translation, or rotation using objects.                      b. Determine whether two polygons are congruent by reflecting, translating, or rotating one polygon to physically fit on top of the other.</p>	<p>Continue with key ideas from previous blocks as needed by students.</p> <p>Continue studying &amp; memorizing basic multiplication facts.</p> <p>Relate division to multiplication in reverse (inverse operations) - beginning with the total items then finding either the <i>number of groups</i> (quotitive, or 'repeated subtraction') or the <i>size of the groups</i> (partitive, or 'sharing'). Generally, students are most familiar with experiences in finding the <i>size</i> of the group.</p> <p>Study division with 2, 3, 4, and 5 as divisors.</p> <hr/> <hr/> <hr/>	<p><b>2008 SFAW Diamond Edition</b>                      7-1 Division as Sharing                      7-2 Division as Repeated Subtraction                      7-3 Writing Division Stories                      7-5 Relating Multiplication and Division                      7-6 Dividing with 2 and 5                      7-7 Dividing with 3 and 4</p> <hr/> <p>8-9 Congruent Figures in Motion</p>
<p><b>Core Vocabulary:</b> divisor, dividend, quotient, reflect, translate, rotate, slide, flip, turn, congruent  <b>Additional Vocabulary:</b> inverse operations</p>		<p><b>Assessment:</b>                      Block 7</p>

**Grade 3 ~ Block 8 ~ Representing Division & Probability**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 3:</u> Model problems involving addition, subtraction, multiplication, and division.                      a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).                      b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.                      c. Demonstrate, using objects, that multiplication and division by the same number are inverse operations (e.g., (e.g., <math>3 \times \square = 12</math> is the same as <math>12 \div 3 = \square</math> and <math>\square = 4</math>).  <u>Objective 4:</u> Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.                      c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).</p>	<p>Continue with key ideas from previous blocks as needed by students.</p> <p>Continue studying &amp; memorizing basic multiplication and division facts.</p> <p>Relate division to multiplication in reverse (inverse operations) - beginning with the total items then finding either the <i>number of groups</i> (quotitive, or 'repeated subtraction') or the <i>size of the groups</i> (partitive, or 'sharing').</p>	<p><b>2008 SFAW Diamond Edition</b>                      7-8 Dividing with 6 and 7                      7-9 Dividing with 8 and 9                      7-10 Dividing with 0 and 1                      7-11 Remainders                      7-12 Division Patterns with 10, 11, and 12</p>
<p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 2:</u> Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.                      a. Represent numerical relationships as expressions, equations, and inequalities.</p>		
<p><b>Standard 5: Students will collect and organize data to make predictions and identify basic concepts of probability.</b>  <u>Objective 2:</u> Identify basic concepts of probability.                      a. Describe the results of events using the terms "certain," "likely," "unlikely," and "impossible."                      b. Conduct simple probability experiments, record possible outcomes systematically, and display results in an organized way (e.g., chart, graph).                      c. Use results of simple probability experiments to describe the likelihood of a specific outcome in the future.</p>	<p>Students make predictions and identify outcomes.</p>	<p>12-7 Describing Chances                      12-8 Fair and Unfair                      12-9 Probability</p>
<p><b>Core Vocabulary:</b> divisor, dividend, quotient, likely, certain, outcome, impossible outcome  <b>Additional Vocabulary:</b> inverse operations</p>		<p><b>Assessment:</b>                      Block 8</p>

**Grade 3 ~ Block 9 ~ Fractions: Equivalence & Comparison**

Core Content	Teaching Strategies	Resources
<p><b>Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.</b>  <u>Objective 2:</u> Use fractions to describe and compare parts of the whole.            a. Identify the denominator of a fraction as the number of equal parts of the unit whole and the numerator of a fraction as the number of equal parts being considered.            b. Define regions and sets of objects as a whole and divide the whole into equal parts using a variety of objects, models, and illustrations.            c. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, sixths, and eighths.            d. Place fractions on the number line and compare and order fractions using models, pictures, the number line, and symbols.            e. Find equivalent fractions using concrete and pictorial representations.</p> <p><b>Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.</b>  <u>Objective 1:</u> Create, represent, and analyze growing patterns.            a. Create and extend growing patterns using objects, numbers, and tables.            b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.</p> <p>** This standard was also in block 1. It is repeated here because students are now exploring growing patterns that have multiplicative structures.</p>	<p>Continue with key ideas from previous blocks as needed by students.</p> <p>Continue studying &amp; memorizing basic multiplication and division facts.</p> <p>Use number line, area/regional, and set models to name and rename fractional amounts.</p> <p>Input/output tables can be used to help students find patterns that are based on multiples.</p>	<p><b>2008 SFAW Diamond Edition</b>            9-1 Equal Parts of a Whole            9-2 Naming Fractional Parts            9-3 Equivalent Fractions            9-4 Comparing and Ordering Fractions            9-5 Estimating Fractional Amounts            9-6 Fractions on the Number Line            9-7 Fractions and Sets            9-8 Finding Fractional Parts of a Set</p>
<p><b>Core Vocabulary:</b> numerator, denominator, halves, thirds, fourths, sixths, eighths, greater than, less than, equal to, &lt;, &gt;, =  <b>Additional Vocabulary:</b> area/regional model, set model, number line model</p>		<p><b>Assessment:</b>            Block 9            Benchmark 3 (reported)</p>

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